MEASURE OF SPREAD: TYPE OF ERRORS THAT STUDENTS MADE IN PROBLEM-SOLVING

Rosidah¹, Fadhil Zil Ikram²
¹Departement of Mathematics, State University of Makassar
²Departement of Mathematics, University of West Sulawesi
¹Jl. Malengkeri Raya No.131, Makassar, Indonesia
²Jl. Prof. Dr. Baharuddin Lopa, S.H, Majene, Indonesia
Email: rosidah@unm.ac.id¹, dhilikram@gmail.com²

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Abstract:
The measure of spread is one of the concepts in Statistics that students find complicated. We have to use a specific formula or concept for a specific situation or contextual problem. As a result, people or students sometimes made mistakes or errors in solving statistics problems. Therefore, research to investigate errors made in solving problems involving a measure of spread is necessary. This study aims to describe students' errors in solving problems related to the measure of dispersion. Researchers used a qualitative descriptive approach. We administered the test during the even semester from January to June in the year 2021. The participants were 80 undergraduate students who have completed or taken statistics courses. This test consists of 2 questions that are related to each other. Researchers used the three main steps to analyze the data which consist of data condensation, data display, and verifying and drawing the conclusion. We obtained the data related to students’ errors from their test results. The results showed that more than 50% of students still make mistakes on every problem. The mistakes made by participants consisted of the use of formulas, miscalculations, the use of known data, the use of concepts, not giving conclusions, and inconsistencies.

Keywords: Undergraduate Students, Errors, Measure of Spread

UKURAN PENYEBARAN DATA: KESALAHAN YANG MAHASISWA LAKUKAN DALAM MEMECAHKAN MASALAH

Abstrak:
Pesertanya adalah 80 mahasiswa yang telah menyelesaikan atau mengambil mata kuliah statistika. Tes ini terdiri dari 2 pertanyaan yang saling terkait. Peneliti menggunakan tiga langkah utama untuk menganalisis data yang terdiri dari kondensasi data, tampilan data, serta memverifikasi dan menarik kesimpulan. Kami memperoleh data terkait kesalahan siswa dari hasil tes mereka. Hasil penelitian menunjukkan bahwa lebih dari 50% siswa masih melakukan kesalahan pada setiap masalah. Kesalahan yang dilakukan oleh peserta terdiri dari penggunaan rumus, kesalahan perhitungan, penggunaan data yang diketahui, penggunaan konsep, tidak memberikan kesimpulan, dan inkonsistensi.

Kata Kunci: Mahasiswa, Kesalahan, Ukuran Penyebaran Data


INTRODUCTION

Nowadays, almost all jobs encounter data or information. These jobs require us to be able to process data or information obtained. Teachers have to analyze the data of their students' progress in many aspects to ensure that they receive the best education. A restaurant owner has to look at their income and trends in the food and beverage business to create a new menu. In this case, statistics play a significant role in our daily life.

In many fields of work, we can see the use of statistics. For instance, survey agencies provide information regarding presidential candidates' electability. In health and medicine, it helps the professionals in a medical study to examine the effectiveness of a specific treatment. Quality testing also involved statistics. For example, companies make sure that they sell the best products by utilizing statistics. Another example is a product owner calculating the mean of the product income or the mode of the product sold to make business-related decisions such as opening a new branch or cooperating with other companies. These illustrations show that statistics is an inseparable part of our daily life.

At all levels of schools and in most university majors, there is at least one course that teaches you about the fundamental of statistics. Additionally, you can find numerous online educational platforms such as edX, Coursera, and Udemy or professional individuals who provide statistics workshops from elementary to expert levels. Some colleges also give online courses in statistics on Youtube or on their website. You can even access a class providing
particular uses of application for statistic analysis. The previous examples and the number of statistics courses available today show how impactful statistics can be.

Statistics is already a prevalent topic that educational experts discuss. The examples were studies by Kaplan, Gabrosek, Curtiss, and Malone (2014); Ji and Kim (2016). The former investigated learners’ comprehension of the histogram and find out that students still struggle even after the instruction. The latter examined students’ conceptual understanding of variability in the data distributions. Jung and Hwang (2016) researched learners’ comprehension of statistical terms, which have lexical ambiguity, for instance, variable, sample, frequency, variance, deviation, etc. There was even a study such as the one by Tam and Chih (2016) analyzing the statistical contents in mathematics textbooks in several countries.

In Indonesia, in general, it is also a common theme that researchers have studied. For example, Rosidah and Ikram (2021) investigated learners' errors in decision-making problems related to the concept of measure of central tendency. Some research analyzed students' difficulties at the level of junior high school (Mediyani & Mahtuum, 2020) and undergraduate (Maysani & Pujiaastuti, 2020). Several studies investigated errors based on several aspects, such as gender (Rahayu & Purwasih, 2020) and learning styles (Fitni, Roza, & Maimunah, 2020). Another example is research that examines students’ conceptual understanding of statistics (Cahani, Effendi, & Munandar, 2021). There was even research discussing the effect of the problem-posing approach to improve learners’ reasoning regarding the concept of measure of central tendency (Chasanah, Sisworo, & Dwiyana, 2019). Another research example is regarding instructional media development. A study by Safitri and Purbaningrum (2020) developed a case-based textbook of statistics.

Despite the number of educational research on the topic, there are still several parts that are rarely explored in depth. One of them is the measure of dispersion. Most of the research discussing the topic only investigates the concept in ungrouped data or just the calculation errors. They did not consider other concepts such as the coefficient of variation or the statistical value of grouped data.

In Indonesia, many students still have poor skills and knowledge in statistics. For example, the percentage of research participants whose scores were below 50 on the test of reasoning in statistics was 68.75% (Chasanah,
Sisworo, & Dwiyana, 2019). Mediyani and Mahtuum (2020) showed that students could answer only 69% of the questions. None of the participants could solve all the problems. Similarly, Fitni, Roza, and Maimunah (2020) reported that out of five questions given, at least students made mistakes in three of them. In addition, a study by Rahayu and Purwasih (2020) even revealed that students’ performance in determining the mean was still in the category of very low. Therefore, students’ skills and knowledge of statistics are still a concern.

Students’ poor skills and knowledge are evident in their test results and homework. Their low performance is usually in the form of mistakes during their problem-solving. These are various types of errors that we could encounter in statistics. For instance, there was a miscalculation due to the numerous formula that learners have to learn or conceptual mistakes because of an incomplete understanding of the statistics concepts. For teachers, listing and analyzing their errors are necessary because they can help the students to overcome their problems. Teachers need a possible explanation of the errors. Based on students’ mistakes, teachers also could design learning that best suits students’ needs, and they can prevent the same mistakes in the future. Thus, knowing and analyzing students’ errors in statistics could be helpful, especially for the learners.

Statistics is an essential part of our life, particularly in the professional world. To prepare students for their desired job, mastery of statistics at university is crucial because this level of education contributes the most to their preparation to achieve their success. However, the previously mentioned research results revealed that many students still possessed poor knowledge and ability in statistics. Moreover, a lack of research on the measure of dispersion is still a concern.

Error analysis is a study analyzing students’ performance to provide a possible description of errors that happened (Herholdt & Sapire, 2014). Based on the background explained, we find it necessary and important to investigate undergraduate students’ errors in solving problems that involve the topic of measure of spread. The results of this research could provide explanations or descriptions regarding the students’ errors. Teachers or instructors in statistics could use the results as a basis to design learning that suits their students’ needs.
METHODS

This qualitative study employed a descriptive approach in explaining the errors that undergraduate mathematics students made when they solved problems regarding the concept of measure of dispersion. We conducted our study during the even semester from January to June in the year 2021. Eighty undergraduate mathematics students participated in this research. The criterion of the participant was that they had studied elementary statistics.

The research instrument utilized was a test consisting of two questions. They are presented in the following table 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The following table are the data of 40 heads of family in the housing area of B income in a week (in the dollar)</td>
</tr>
<tr>
<td></td>
<td>Class</td>
</tr>
<tr>
<td>1</td>
<td>125 – 134</td>
</tr>
<tr>
<td>2</td>
<td>135 – 144</td>
</tr>
<tr>
<td>3</td>
<td>145 – 154</td>
</tr>
<tr>
<td>4</td>
<td>155 – 164</td>
</tr>
<tr>
<td>5</td>
<td>165 – 174</td>
</tr>
<tr>
<td>6</td>
<td>175 – 184</td>
</tr>
</tbody>
</table>

Based on the data, determine its variance and standard deviation!

2. If the mean income of 40 family heads in the housing area of C (in the dollar) is 38 with the variance of 88.36. Is the data in number 1 more homogeneous?

Students’ answers or responses are the sources of the data. We obtained it from their test results. We analyzed students’ results by following the three main steps proposed by Miles, Huberman, and Saldana (2014). They are data condensation, data display, and drawing conclusions and verification.

The analysis of the responses began by making two categories, which are correct and incorrect for each question. In the latter category, we grouped the responses which had the same mistakes. After forming the class, we selected the responses that represented the group by comparing which one was the most suitable or the one that was the most distinct compared to other responses.
In data display, our study used figures to describe the selected answers and employed tables to show the distribution of the categories. We used them to find a pattern or a unique finding by comparing figures selected or between the figures and the tables.

Based on the results obtained, we made conclusions regarding undergraduate students’ errors in solving problems involving the concept of measure of spread. Finally, as part of the third stage of data analysis, verifying our conclusion is also necessary. We compared our findings to look for things that we might miss. The process was repeated until we reached conclusions.

RESULTS AND DISCUSSION

1. Results

After administering the test and analyzing the participants’ answers, the following table 2 provides information about the number of students’ responses that fall in each category for each question.

<table>
<thead>
<tr>
<th>Question</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct</td>
</tr>
<tr>
<td>1</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 2 shows that many undergraduate students still failed to achieve a full score on the test. More than half of the students did not answer the question correctly. Moreover, in the second question, the number of undergraduate students who gave incorrect responses was twice as many as the number of participants who answered correctly.

Many participants did not respond at all to the questions given to them. Seventeen undergraduate students did not write anything on their answer sheets in question 2. However, in question 1, only 2 participants whose response sheets were blank. Additionally, one and five students did not finish their answers to the first and second questions, respectively. There was even a participant who just wrote the final result without providing the process.

Among the students’ answers, we found many kinds of mistakes, for example, miscalculation, the use of the wrong formula, and other errors. It should be noted that some participants made more than one error. The next section discusses every error found in our study.
a. Use of The Wrong Formula

The most common error that participants made in the test were using the wrong formula. Twenty students made these errors. One of the incorrect formulas they used did not include the multiplication of the class frequency. The following figure illustrates such an error.

![Example of Wrong Formula](image1.png)

Figure 1. Example of Wrong Formula

Figure 1 reveals that they did not multiply the \((x_i - \bar{x})^2\) with the frequency of each class. Moreover, they used \(n = 40\) instead of \(n = 39\). These two errors lead to a wrong result making them fail to gain a full score on each question. These participants employed the formula of ungrouped data instead of the grouped one.

b. Miscalculation

Calculation error was one of the mistakes that happened the most in our study. In question 1, 18 undergraduate students miscalculate the standard deviation and variance values. The following figure 2 presents one example of miscalculation.

![Example 1 of Miscalculation](image2.png)

Figure 2. Example 1 of Miscalculation

In calculating the standard deviation value of grouped data, the participants should calculate the difference between the middle value of each
class and the mean, find the square of the result, and then multiply it by the class frequency. The result should be 1.584,375, but one of the participants wrote 2.584,375. Another case is shown in figure 3.

\[ s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} \]
\[ = \sqrt{\frac{10537.5}{40}} \]
\[ = 2.63,4375 \]

Figure 3. Example 2 of Miscalculation

Figure 3 shows that the participant of the test might forget to calculate the square root of \( \frac{10537.5}{40} \). Instead, the student just determines the result of the division. Figure 3 also reveals another error, which is the use of the wrong formula. In question 2, 13 students made a calculation error due to their mistakes in the first question. Some of the participants who made the error in question 1 eventually miscalculated because the two questions are related.

c. Used the Wrong Data

Several undergraduate students used the correct formula to determine the variance coefficient in the second question. However, some of them utilized the wrong data. Figure 4 shows the error.

Figure 4. Example of Utilization of Wrong Data
Instead of using standard deviation, participants determined the coefficient by dividing the variance by the mean. They likely thought the term “Variation” in coefficient of variation signifies that we have to use the variance itself.

d. Inappropriate Use of Concept or Use the Wrong Concept

Ten out of the eighty responses used the concept of variance and mean inappropriately or used the wrong one to answer question 2. Some participants compared the variance and mean of the two sets of data to draw their conclusion, but they did not use the coefficient of variation to ensure their decision. Such errors are presented in figure 5.

![Figure 5. Example of Inappropriate Use of Concept](image)

Figure 5 shows that the participant already used mean and variance to compare which one of the data sets is more homogeneous. However, they did not employ the coefficient of variation to answer such a question. In addition, their use of mean and variance is incomplete. They just compared which one is higher. Using the variance is correct, but you have to calculate the coefficient to ensure the correctness of your answers. Moreover, they even mentioned the frequency and also did the comparison.

The next error made by the students was utilizing the wrong idea. The following figure 6 illustrates such an error.
Figure 6. Example of the Use of Wrong Concept

Based on figure 6, the participant thought that the data of Income in Housing B is more homogeneous than the Housing A because its variance and mean are more detailed and accurate. The student might view that the number of numbers after the comma means that it is more accurate.

e. No Conclusion

There was a participant who almost achieve a full score on the test. Nevertheless, the conclusion is not given. Figure 7 shows such an error.

Figure 7. Absence of Conclusion

The figure reveals that there is a final answer to the second question. The item asked us to decide which one of the data sets is more homogeneous than the other by calculating the coefficient of variation. However, in Figure 7,
the participant only calculates the coefficient without deciding which one is more homogeneous. Therefore, we cannot give the student the maximum score because the conclusion is also an inseparable part of the answer.

f. Inconsistencies

The last error found in the test was inconsistency. Figure 8 shows an example of such an error.

![Figure 8. Example 1 of Inconsistencies](image)

The figure reveals that the participant employed the wrong formula to determine the standard deviation and variance of the data, which was dividing by \( n \) (40) instead of \( n - 1 \) (39). The response to the second question also did not match the answer to the first one. The standard deviation of the data in question 1 is 16.44. However, this answer is the result of dividing by \( n - 1 \), which is 39. Moreover, the participant did not finish their calculation.

Another example of inconsistency that happened during the test is evident in Student B’s Answer. The results are shown in figure 9 and figure 10 below.

![Figure 9. Student B’s Answer to Question 1](image)
The two figures clearly show the inconsistency that students made during the test. In the first item, they wrote that the standard deviation of the data is 666,979. Nonetheless, the participant answered that the standard deviation is 16,44 in the second question. The case is similar to the response in figure 8, and there is two possible explanation for it.

2. Discussion

Students who leave the answer blanks or only list the data needed to answer the questions might make a transformation error. They did not know the necessary operation to solve the problems and the appropriate procedures to execute the process ((Ken) Clements, 1980; White, 2005). Thus, they cannot continue to finish the succeeding steps to solve the questions.

Rahayu and Purwasih (2020) conducted a survey to analyze students’ errors in statistics, particularly junior high school students. They reported that only 8% of their students could determine the range of the data correctly. As we already know, the range is one of the values in measures of dispersion that is easy to calculate. The study provides evidence that students struggle with the concept of spread from an early level.

The next paragraphs discuss the other errors students made during the test. The errors are using the wrong formula, miscalculation, using the wrong data, inappropriate use of the concept, lack of conclusion, and inconsistencies.

According to Newmann Theory, utilizing the wrong formula could be categorized as a process skills error. The mistake happened because an
individual does not know the right procedures to carry out the operations needed to answer the question ((Ken) Clements, 1980; White, 2005). The participants knew what kind of value that must be calculated, but they did not know the correct formula to find such value.

Using the wrong formula is an error found in many studies. Research by Maysani and Pujiastuti (2020) described that 10% of their students used incorrect procedures to determine statistical values such as median and variance. Undergraduate students who struggle in using formulas were also described in a study reporting that they have conceptual and procedural errors concerning the formula used in the measure of spread (Wahyudi, Abadyo, & Purwanto, 2017).

Miscalculation is one of the errors that are usually found in students’ responses. Figure 2 shows an example of a careless error ((Ken) Clements, 1980) that might happen due to a miscalculation. Undergraduate students miscalculated statistical values because they do not thoroughly check their answers before submitting them to the teachers. The long process and the number of formula and calculation that has to be carried out might be one of the factors of miscalculation.

The participant error, in figure 3 could be called a basic error. The miscalculation that did not find the square root of \( \frac{10537.5}{40} \) is an error that you might not expect to encounter at a certain level (Brodie & Coetzee, 2010). At the university level, students should already understand that \( \sqrt{\frac{10537.5}{40}} \) means that we have to find the results of the division and find the square root of it. However, the participant did not calculate the square root and just carried out the division. Students might forget about it or just not know about it.

Similar findings by a study also revealed that 30% of the students in the course of descriptive statistics made miscalculations (Maysani & Pujiastuti, 2020). Novalia (2019) also reported that most of the students in the statistics course made a miscalculation error. It shows that the problem of calculation remains in many parts of statistics.

Using the wrong data or using concepts inappropriately could be called concept errors. It happens when people do not comprehend the properties required to solve or answer problems (Nolting, 2012). In the case of Figure 5 and Figure 6, students did not understand that they need to calculate the coefficient of variation to validate their answers. So, standard deviation, variance, and mean are not enough. For example, it is easier if two data sets...
have the same mean values with different standard deviations. However, for instance, one set of data has a mean of 29 and a standard deviation of 10, and the other one has a mean of 950 and a standard deviation of 10. The deviation of the former is considered high while the latter is not. To ensure and prove it, calculating the coefficient of variation is one of the ways.

Students’ mistakes that did not provide a conclusion could be categorized as test-taking errors, particularly the error of not completing or finishing the last step of problem-solving (Nolting, 2012). Such mistake happens because of individuals who do not review their works. Reviewing every step and process of our work is necessary to ensure that there is no error in our answers. The step of reviewing is also needed to avoid miscalculation errors.

The inconsistencies might be called test-taking errors because of miscopying answers (Nolting, 2012). Students might find out that their initial responses are wrong and then try to solve the problem again on scratch paper. However, they copied the false one to the test leading to an incorrect final result. An error such as inconsistency was also described by some researchers. A study reported that 4 out of 10 students were inconsistent in using mathematics symbols in solving questions in statistics (Maysani & Pujiastuti, 2020).

There are two other possible explanations as to why there were inconsistent responses. Firstly, the student prepared two answer sheets. The first one is for trial and error, and the other one would be given to the teacher. Students might realize that their answers were incorrect but forgot to move the results from the former sheets to the latter one, and they already ran out of time. Secondly, there was a possibility of cheating. Although this is the worst scenario, it might happen. Due to running out of time, the participant just copied their friends’ results without looking carefully at the answer sheets.

The findings of our study have several implications. Teachers or instructors could use them as the basis to design learning on the concept of measure of spread that suits students’ needs. They also could develop media or tools to tackle students’ problems based on the explanation of our results. Other researchers also could utilize our findings to create a new categorization of students’ errors in solving statistics problems.
CONCLUSION

The findings of this research reveal that there are still numerous undergraduate students who have low performance in statistics, especially on the topic of measure of spread. They make several errors such as using the wrong formula, miscalculation, using the wrong data or statistic, inappropriate use of a concept or using the incorrect one, absences of conclusions, and inconsistencies. Some of the mistakes even indicate the possibility of cheating behavior. Based on these results, other researchers interested in a similar topic could explore more by addressing another part of the measure of spread. Other topics such as the design of learning on the concept of measure of dispersion or developing instructional tools of the concept are also recommended.

REFERENCES


