ANALYSIS OF 8 GRADER STUDENTS’ PERFORMANCE AT SMP AM IN UNDERSTANDING THE TRIANGLE CONCEPT BASED ON NCTM GEOMETRY STANDARD

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Abstract:
Geometry plays a pivotal role in mathematics education, contributing significantly to the development of students' spatial awareness, intuition, and visualization skills. Despite its importance, there are reported challenges in students' understanding of geometry, with the Merdeka curriculum geometry standard noted for not adequately emphasizing the fundamental understanding of triangle properties before introducing congruence. Conversely, studies have highlighted the effectiveness of incorporating NCTM (National Council of Teachers of Mathematics) standards in fostering mathematical comprehension in this domain. This research study delves into the performance of eighth-grade students in comprehending triangle concepts, guided by the NCTM geometry standard. Employing a content analysis technique on student work, created based on NCTM indicators and assessed accordingly, the study reveals deficiencies in students' understanding. These challenges include difficulties in describing and categorizing triangles, identifying angle relationships, constructing logical arguments using both inductive and deductive reasoning, and recognizing relationships between side lengths. These findings underscore the urgent need to address these knowledge gaps and bridge the disparity between NCTM standards and students' current comprehension levels regarding triangle concepts.

Keywords: Students Profiles, Triangle Concept, NCTM, Content Analysis.
Council of Teachers of Mathematics) dalam meningkatkan pemahaman matematis di bidang ini. Studi penelitian ini menyelidiki kinerja siswa kelas delapan dalam memahami konsep segitiga, dipandu oleh standar geometri NCTM. Dengan menggunakan teknik analisis konten pada karya siswa, yang dibuat berdasarkan indikator NCTM dan dinilai sesuai, studi ini mengungkap kekurangan dalam pemahaman siswa. Tantangan ini melibatkan kesulitan dalam menggambarkan dan mengategorikan segitiga, mengidentifikasi hubungan sudut, membangun argumen logis menggunakan penalaran induktif dan deduktif, serta mengenali hubungan antara panjang sisi. Temuan ini menekankan kebutuhan mendesak untuk mengatasi kesenjangan pengetahuan ini dan menjembatani kesenjangan antara standar NCTM dan tingkat pemahaman siswa saat ini mengenai konsep segitiga.

Kata Kunci: Profil siswa, Konsep Segitiga, NCTM, Analisis Konten.


INTRODUCTION

Geometry is an important learning subject in mathematics that enhances students' spatial awareness, intuition, and visualizations of shapes and their properties (Fabiyi, 2017). It plays a crucial role in enabling students to analyze and interpret the world around them, with applications in various fields, including the study of the solar system (Volderman, 1998; Fabiyi, 2017). Despite its importance, many students struggle with geometry concepts, particularly triangles.

Several studies have identified common difficulties faced by students in understanding angles, triangles, and their relationships. Research conducted by Fabiyi (2017) which focuses on eight geometry concepts found that the mistakes students make are in understanding the problem information, not solving the problem systematically, and not understanding the representation. Sehatta (2002) whose research on seventh-grade junior high school students found that, in general, students did not have strong knowledge of the characteristics of triangles, making it difficult for them to categorize a triangular object, in this case, the classification of the same type of triangle's feet, equilaterals, and right angles. In other words, students are unaware of how the idea of an equilateral, isosceles, or right triangle can be defined. A study conducted by Biber (2020) that studied students’ difficulties in learning
triangles, indicates that students have difficulties mostly in overlapped triangles and angle-angle type of questions. On the other hand, they are quite successful when similar triangles are given separately. There should be a bridging of understanding from a separately given triangle to a composed-overlapped triangle.

This shows that understanding the triangle concept is still an issue for students. A study conducted by Kusno and Sutarto (2022) found that the most common errors among students occurred in their understanding of angles and triangles. They mistakenly assumed that an angle was simply a point or a portion of a plane between two lines, and they believed that a triangle was a part of the plane formed by three-line segments. These errors were primarily caused by a lack of understanding of the fundamental concepts. Students were unfamiliar with the undefined terms, the role of rays and line segments in defining angles and triangles, and the functions they served. As a result, they struggled with interpreting conceptual images, reasoning, and making connections between the necessary knowledge to draw, construct, and define angles and triangles accurately. Another study from Iskak, Kusmayati, and Fitriana (2020), also said that students continued to struggle with the process of translating triangle problems into suitable visual, verbal, and symbolic representations for solving them effectively. Lastly, a study conducted by Lutfi, Juandi, and Jupri (2021) found the students made an error in identifying the base and height lines of the triangle. This indicates a lack of mastery of the concept of base and height in triangles. The mistake was due to a technical error made by the students themselves. While this error may seem insignificant, it can pose difficulties for students if not anticipated by the teacher. From those studies, it can be concluded that most students have difficulty when it comes to angle relationships, interpreting conceptual images, reasoning, and making connections, and representations. Kurikulum Merdeka, the current curriculum implemented in most schools in Indonesia, emphasizes the importance of knowing about angle properties and utilizing angle relationships. However, it has not been stated to emphasize this understanding and connect to the triangle properties understanding, before using it in relationships of triangles. These are the indicators for geometry subject in the schoolbook which is taken from the kurikulum merdeka (Budi, 2022): (1) Students should be able to create geometric nets and convert them back into their original shapes. (2) Students are expected to understand and utilize angle relationships. (3) Students should be able to explain the properties
of congruence in triangles and quadrilaterals. (4) Students are expected to demonstrate the validity of the Pythagorean theorem. (5) Students should be capable of performing a single transformation of points, lines, and plane shapes on the Cartesian coordinate.

From those 5 points above, it is not clear whether point no 2 gives any basic idea to understand point 3. Students see it exclusively, not having any relationship.

To address this challenge, aligning the Kurikulum Merdeka with the NCTM standards can enhance math education in Indonesia. The NCTM standards promote a deep understanding of math concepts and reasoning and can provide valuable insights into students' performance in geometry (Suyitno, Utami, & Veronica, 2019; Joung & Byun, 2021). Analysis of the student's work using the NCTM indicator will be valuable.

This research aims to explore the 8th-grade students' performance in understanding the concept of triangles, aligning with the NCTM geometry standards. The study's objectives are to determine the overall level of understanding of triangles among the students and identify specific areas of strength and weakness in their comprehension of triangles based on the NCTM standards. The research is expected to offer valuable insights into students' understanding of triangles and can help improve teaching strategies and curriculum development to enhance learning outcomes in geometry education.

**METHODS**

This study aims to understand how students comprehend the concept of triangles based on the NCTM indicator. In achieving the research aims, the Qualitative research design will be employed. According to (Creswell, 2012), to explore certain phenomena, which as in this study, the students' understanding of triangle properties, is better to use qualitative study. The researchers believe that qualitative research methods are more suitable for this purpose. To achieve their goals, they used a content analysis approach. Content analysis, as described by Krippendorff (2004), involves analyzing relevant texts or materials within their respective contexts to draw reliable and valid conclusions. In this study, the data analyzed likely consists of students' responses. Stemler (2001) defines content analysis as a systematic research method that identifies specific message characteristics to make objective inferences. The research focuses on assessing students' mathematical
understanding when solving geometric problems, specifically those related to triangles, following NCTM standards.

This study examined a group of 8th-grade students at SMP AM, one of the private schools in Jakarta. The researchers along with the mathematics teachers selected students from three different grade categories (good, fair,
and poor) to create a diverse background of the research subject. The school math teacher played an important role in the selection process, intentionally choosing students with varying levels of performance and understanding of the triangle concept. By using this purposive sampling technique, the researchers ensured a comprehensive analysis aligned with the research goals, allowing them to examine the students' performance in the NCTM standard. This technique, as described by Fraenkel and Wallen (2008), allows the researchers to ensure that the research subject represents a diverse range of abilities and levels of understanding.

A study was conducted to assess students' understanding of geometry concepts, particularly related to triangles. The researchers developed a set of questions based on the NCTM Geometry Standard for grades 6-8. To ensure the accuracy and relevance of the questions, two mathematics lecturers and the school's mathematics teacher evaluated them. These validators had specialized knowledge in mathematics and were well-suited for this task. Through a rigorous validation process and necessary revisions, the questions were deemed reliable and valid for assessing students' comprehension of geometry and their understanding of triangles. The specific assessments used in the study can be found in Table 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Problem</th>
</tr>
</thead>
</table>


The study employed a descriptive qualitative data analysis approach to comprehensively analyze the students' responses to the assessment. Descriptive qualitative analysis is a naturalistic research method that involves straightforward interpretation and presents information in everyday language.
The analysis aimed to describe the existing conditions and categorize the data based on indicators derived from the NCTM geometry standards for grades 6 and 8. The indicator used in this research is presented in Table 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>a) Students can describe triangles using their defining properties</td>
</tr>
<tr>
<td></td>
<td>b) Students can classify triangles using their defining properties</td>
</tr>
<tr>
<td></td>
<td>c) Students can identify the relationship among the angles of triangles</td>
</tr>
</tbody>
</table>

### RESULTS AND DISCUSSION

Out of the total target population of 17 students, a deliberate selection process was employed to choose 6 students, with representation from the categories of good, fair, and poor grades. This purposive sampling technique involved the selection of 2 students from each category. The specific codes assigned to these students can be found in Table 3. The analysis of these students' work was conducted using indicators 1a to 1c, aiming to derive a comprehensive profile of their understanding of triangle properties.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Students Code</th>
</tr>
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<tbody>
<tr>
<td>Good</td>
<td>G1</td>
</tr>
<tr>
<td></td>
<td>G2</td>
</tr>
<tr>
<td>Fair</td>
<td>F1</td>
</tr>
<tr>
<td></td>
<td>F2</td>
</tr>
<tr>
<td>Poor</td>
<td>P1</td>
</tr>
<tr>
<td></td>
<td>P2</td>
</tr>
</tbody>
</table>

1. **Indicator 1a: Students can describe triangles using their defining properties.**

This indicator emphasizes the importance of understanding the properties of triangles. It includes having three sides and three angles, with the sum of the angles always being 180 degrees. Triangles can be categorized based on the length of their sides and the measure of their angles. Students should be capable of identifying the number of sides and angles, determining the sum of the angles, and classifying triangles based on their side lengths and angle measures. It was found that most of the students had difficulty describing:
a. **Isosceles triangle when one of the angles has 120 degrees**

An isosceles triangle is characterized by two equal sides and two equal angles at its base, with the total sum of angles always equating to 180 degrees. Unfortunately, among the six students assessed, five demonstrated a lack of recognition of the possibility of an isosceles triangle. Their explanations were inaccurate, with some mistakenly believing that isosceles triangles have angles below 90 degrees or possess a plane less than 120 degrees. Additionally, misconceptions included the belief that isosceles triangles cannot be slanted or that their sides should measure 120 degrees. These students lacked a fundamental understanding of isosceles triangle properties. However, a notable exception was observed in one student, G2, who correctly comprehended that an isosceles triangle can have one angle measuring 120 degrees. This indicates G2's awareness of the triangle's properties, as depicted in the accompanying picture. The drawing portrayed one angle as an obtuse angle, while the other side of the triangle exhibited equal lengths. Nevertheless, it is suggested that G2 specify the possible range of angles for the triangle for a more comprehensive understanding.

![Figure 2. Student response in describe isosceles triangle when one of the angles has 120 degrees (left: correct, right: incorrect).](image)

b. **Scalene triangle when one of the angles has 60 degrees**

A scalene triangle is a triangle with three sides of different lengths and no equal angles or sides. The sum of the angles in a scalene triangle is always 180 degrees. It's interesting to note that the lengths of the sides and the measures of the angles are not directly related. One student, G1, observed that
a scalene triangle can have an obtuse angle. Another student, P2, correctly understood that a scalene triangle can have one side measuring less than 90 degrees, but her explanation lacked detail. Out of the six students who answered correctly, three of them knew that a scalene triangle can have an angle of 60 degrees.

![Figure 3](image)

Figure 3. Student response in describe scalene triangle when one of the angles has 60 degrees (left: correct, right: half correct)

c. Right-angled triangle when one of the angles has 60 degrees

A right triangle has specific properties. It has one angle that measures exactly 90 degrees, and the longest side, called the hypotenuse, is opposite the right angle. The other two sides are known as the legs. The sum of angles in a triangle is always 180 degrees. Among the students, one named F1 correctly recognized that all right angles measure 90 degrees, showing an understanding of right triangles. However, out of the six students who answered correctly, two failed to explain how a right triangle can have an angle measuring 60 degrees. Student P2's explanation lacked details and did not include a triangle diagram, making it unclear which angle she was referring to as less than 60 degrees. This suggests that she is not familiar with the properties of a right triangle, which include one angle being 90 degrees and the total sum of angles being 180 degrees.
d. **Obtuse triangle when one of the angles has 60 degrees**

An obtuse triangle is a type of triangle that has one angle greater than 90 degrees, while the other two angles are less than 90 degrees. The longest side of the triangle is the one opposite the obtuse angle. The sum of all angles in an obtuse triangle is always 180 degrees. However, when presented with an obtuse triangle where one of the angles is 60 degrees, none of the students were able to accurately describe it. The illustration is given in the figure 5.

![Figure 4](image1.png)

**Figure 4.** Student response in describe right triangle when one of the angles has 60 degrees (left: half correct, right: incorrect)

![Figure 5](image2.png)

**Figure 5.** Student response in describe right triangle when one of the angles has 60 degrees (left to right: incorrect).
2. **Indicator 1b: Students can classify triangles using their defining properties.**

Students can be said to fulfill this indicator if they can:

a. Categories triangles based on the angles.
   - Acute triangles: all angles are less than 90 degrees.
   - Right triangle: one angle is exactly 90 degrees.
   - Obtuse triangle: one angle is greater than 90 degrees.

b. Categories triangles based on the side lengths.
   - Equilateral triangle: all sides are equal in length.
   - Isosceles triangle: two sides are of equal length.
   - Scalene triangle: all sides have different lengths.

The findings of this indicator will be categorized into two parts, based on the length of the sides and based on the angle measurement.

![Figure 6. Student response in classify triangles based on the length (left: one of the angles has 120 degrees, right: one of the angles has 60 degrees)](image)

c. Based on the length of the sides

Almost all students have difficulty classifying triangles based on their side lengths. They struggle particularly with identifying isosceles and scalene triangles when one angle measures 120 degrees. Instead of considering side lengths, they primarily focus on angle sizes to classify triangles. The student's tendency to rely on angle measurements rather than side lengths indicates possible confusion or lack of knowledge in this specific area.

![Figure 7. Student response which indicates focusing only on the angle properties, not considering the length side](image)
One of the interesting cases is the case of G1. G1 seems to have an understanding that when the isosceles triangle has only an acute angle, and when one of the angles is 60 degrees it will be creating a right triangle. This understanding will limit students' observation to the side angle, hindering them from seeing the other possibilities.

The other misunderstanding about triangle properties is also seen in F1. She mentioned that isosceles and equilateral triangles are the same, just because of the word “sama”. Clearly, the student does not understand the properties of a triangle in terms of side length.

Moreover, the above illustrations give insight that students have not yet fully understood the properties of triangles in terms of side length. The given information is about the angle, however, the students also cannot see the relationship and implication of the angle magnitude to the side length properties.

d. Based on the size of the angles

Most of the students know what it means by an obtuse angle, acute angle, and right angle. However, students cannot take the relationship between the type of angle and the type of triangle with the same name. They define the name of a triangle just because there is a presence of a certain type of angle. For example, right triangle, they only focus on one angle instead of seeing the three angles in a triangle as one unity. Since the given information is a 60-degree angle, is it possible to have a right triangle the student said no as 60 degrees is not the right angle. The student work is illustrated below in figure 9.
Analysis of 8 Grader Students’ Performance…

Figure 9. Student response in classify triangles based on the size of the angles (left: one of the angles has 120 degrees, right: one of the angles had 60 degrees)

Furthermore, a student is incorrectly labeled a triangle with a 60-degree angle as obtuse, indicating a misconception about obtuse angles. They correctly identified a right triangle with a 90-degree angle, however, the student ignored the other angle that forms the right triangle and explained that a 120-degree angle cannot be a right angle.

Finally, no students are using the fundamental properties of angle measurement in a triangle. Students are not indicating the sum of angles in giving the reason why an angle of 120 degrees cannot form a right triangle. They focus only on the given information (one of the measures of angle in a triangle), and directly give the conclusion.

Figure 10. Student response in classify triangles based on the size of the angles (left: one of the angles has 120 degrees, right: one of the angles had 60 degrees)

3. **Indicator 1c: Students can identify the relationship among the angles of a triangle.**

In this indicator, students said as able to identify relationships among angles of a triangle if they know that:
a. The sum of the interior angles in any triangle is always 180 degrees.
b. The base angles of an isosceles triangle are congruent.
c. The angles of an equilateral triangle are all congruent and measure 60 degrees.
d. If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.

After being analyzed, it was found two cases are interesting to be taken into further detail explanations. Student F1’s work is depicted in the figure 11.

Figure 11. Student F1 work (left: one of the angles has 120 degrees, right: one of the angles had 60 degrees)

Student F1 faces challenges when it comes to recognizing specific types of triangles based on their angle relationships. Specifically, she has difficulty identifying right triangles and obtuse triangles when one of the angles measures 60 degrees. Her understanding of the angle relationships within triangles seems to be limited, as her explanations do not fully demonstrate a comprehensive awareness of these relationships. The illustrations also add to the inaccuracies, as she incorrectly depicts a right triangle as an acute triangle and does not include complete angle measurements.

Figure 12. Student P2 work (left: one of the angles has 120 degrees, right: one of the angles had 60 degrees)
According to student P2’s response, her understanding of identifying different types of triangles based on angle relationships is incomplete. She provided incorrect answers when dealing with isosceles triangles having a 120-degree angle and obtuse triangles with a 60-degree angle. Although she demonstrates some knowledge in recognizing other triangle types, her explanations lack the necessary depth to show a strong comprehension of angle relationships within triangles. While she correctly describes an equilateral triangle as having equal side lengths, it is unclear whether she knows that the angles in an equilateral triangle are also equal. Student P2 shows some ability in identifying certain triangle types but struggles with isosceles triangles having a 120-degree angle and obtuse triangles with a 60-degree angle. Additionally, her response doesn’t confirm her understanding of equal angle measurements within equilateral triangles.

This research aimed to assess the alignment of 8th-grade students’ understanding of triangle concepts at SMP AM with the NCTM geometry standard. The analysis of the assessment results revealed notable shortcomings in the student's performance across several key areas.

Firstly, challenges emerged in accurately describing triangles using their defining properties, particularly in the case of isosceles triangles with a 120-degree angle. Misconceptions and a lack of understanding regarding the properties of isosceles triangles were evident. Similar difficulties were observed in handling right-angled triangles and obtuse triangles with a 60-degree angle, showcasing confusion and incorrect assumptions related to acute angles, obtuse angles, and side lengths. This echoes findings from Siregih and Sehatta's (2022) research, emphasizing students' struggles in defining the sides and angles of various triangles.

Secondly, students faced issues in classifying triangles based on their defining properties, displaying errors and unclear explanations when identifying isosceles, equilateral, and scalene triangles. The accurate classification of triangles based on both side lengths and angle measurements posed challenges, as illustrated by incomplete or inaccurate representations in their work.

Thirdly, a majority of students struggled to identify the relationships among the angles of triangles, exhibiting limited or no ability to correctly identify and classify different types of triangles based on their angle relationships. Responses lacked sound reasoning, accurate explanations, and
appropriate visual representations, aligning with findings from Sarama and Clements (2004), which highlighted difficulties in triangle identification.

Moreover, challenges were observed in creating inductive arguments concerning geometric ideas and relationships among types of triangles. While some students displayed relative proficiency in creating inductive arguments, weaknesses were evident, attributed to a lack of familiarity with undefined terms and a limited understanding of fundamental triangle concepts, in line with Kusno and Sutarto's (2022) research.

Additionally, students faced difficulties in recognizing relationships among side lengths of triangles, leading to errors in identifying these relationships, particularly in the application of the Pythagorean theorem.

Lastly, the creation of deductive arguments and recognition of relationships among triangles, such as congruence, posed challenges for many students. While some demonstrated proficiency in inductive arguments, their deductive reasoning fell short of expectations.

The research underscored that students at SMP AM cannot meet all assessed indicators about triangle properties. Significant gaps were identified in their understanding of the relationship between angle properties to the type of triangle. Notably, these gaps were reflected in the "Kurikulum Merdeka" used by SMP AM, where the indicator of explaining the properties of congruence in triangles and quadrilaterals remained unmet by the students.

In conclusion, the study emphasized the pressing need to address the disparity between the desired outcomes outlined by the NCTM standards and the current understanding of triangle concepts among students.

CONCLUSION

This research highlights significant gaps in the understanding of triangle concepts among 8th-grade students at SMP AM about the NCTM geometry standard. The findings reveal that students face difficulties in accurately describing and classifying triangles, identifying angle relationships, creating logical arguments, and recognizing relationships among side lengths. This study emphasizes the importance of addressing these gaps and bridging the disparity between the desired outcomes set by the NCTM standards and the student's current understanding of triangle concepts.

The performance of the 8th-grade students at SMP AM regarding triangle concepts, based on the NCTM geometry standards, can be concluded as not yet meeting all the standards. The specific challenges identified are as
follows: (1) Students struggle to describe specific types of triangles due to a lack of knowledge about their properties. (2) Students have difficulty classifying triangles based on side lengths and angle measurements, often presenting incorrect arguments and incomplete illustrations. (3) Students find it challenging to identify angle relationships in triangles due to insufficient supporting arguments and mismatched illustrations. (4) Students face difficulties in creating inductive arguments about geometric ideas and relationships among different types of triangles due to a lack of foundational knowledge.

The findings suggest a potential misalignment between the NCTM standards and the "Kurikulum Merdeka" curriculum used by SMP AM. While the NCTM standards emphasize the importance of inductive and deductive reasoning in congruence by firstly understanding the relationship between the angle properties and triangle properties, the school's curriculum does not explicitly include these aspects. This misalignment should be addressed to ensure that the curriculum adequately prepares students to meet the standards and develop a deeper understanding of geometric concepts.

Based on the research findings, the researcher suggests that 8th-grade students at SMP AM review triangle materials from 7th grade or earlier to strengthen their understanding of triangle concepts and practice them through exercises. Additionally, mathematics teachers are advised to thoroughly discuss the concept of triangles in the mathematics subject by assigning tasks or exercises that enhance learning, encourage students to make connections between different aspects of a problem, and improve their ability to analyze geometric information effectively.

Furthermore, it is recommended to conduct further studies to investigate the teaching and learning activities inside the classroom. Specifically, these studies should explore instructional practices that can enhance students' performance when learning about the triangle concept, considering various factors that can influence student performance.

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