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DEFRAGMENTATION OF CONSTRUCTION HOLE THINKING PROCESS OF JUNIOR HIGH SCHOOL STUDENTS IN SOLVING NUMERATION LITERACY QUESTIONS

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Abstract:
Teachers must be aware of and take action regarding students' errors in creating incomplete mathematical concepts, also referred to as construction holes, particularly when it comes to answering numeracy literacy questions. This will help students rebuild their understanding of numeracy literacy questions overall. By using diagrams to depict the answers to numeracy literacy questions both before and after the defragmentation process, this study seeks to identify the places where pupils fall short conceptually. A mixed-methods research methodology was employed for this study, which was conducted in two phases: quantitative analysis was used in the first phase, and qualitative analysis in the second. Based on the research findings, the numeracy component of the statistics curriculum is where the gaps in students' thinking processes are located. Students' thought structures can be strengthened by the use of defragmentation. When answering questions on numeracy and literacy, the defragmentation method utilized is scaffolding at level two, which consists of explaining, reviewing, and restructuring. The mean, mode, and median material in the student results contained construction holes that were later resolved through a scaffolding type that involved three steps: explaining, reviewing, and restructuring. This process helped students avoid making mistakes when answering the questions about the mean, mode, and median.

Keywords: Defragmentation, Construction Holes, Literacy in Numeracy, Cognitive Processes

DEFRAGMENTASI LUBANG KONSTRUKSI PROSES BERPIKIR SISWA SMP DALAM MENYELESAIKAN SOAL LITERASI NUMERASI

Abstrak:
Kesalahan siswa dalam mengonstruksi konsep matematika yang tidak secara utuh atau disebut dengan lubang konstruksi, khususnya dalam menyelesaikan soal literasi numerasi merupakan hal yang perlu diketahui dan ditindaklanjuti oleh guru sehingga siswa bisa kembali membangun pengetahuan tentang soal literasi numerasi secara utuh. Penelitian ini bertujuan untuk mengetahui letak konstruksi konsep siswa yang tidak secara utuh dalam menyelesaikan soal literasi numerasi yang divisualisasikan ke dalam diagram sebelum dan sesudah proses defragmentasi.

Penelitian ini menggunakan desain penelitian mixed method yang dilakukan secara dua tahap yaitu tahap pertama menggunakan analisis kuantitatif, dan tahap kedua menggunakan analisis kualitatif. Hasil penelitian yaitu letak lubang konstruksi struktur berpikir siswa terjadi pada komponen numerasi pada materi statistika. Defragmentasi merupakan metode untuk memperbaiki struktur berpikir siswa. Defragmentasi yang digunakan yaitu scaffolding pada level dua yaitu (explaining, reviewing, dan restructuring) dalam menyelesaikan soal literasi numerasi. hasil yang diperoleh siswa mengalami lubang konstruksi materi mean modus dan median yang kemudian didefragmentasi menggunakan tipe scaffolding yang terdiri dari tiga tahapan yaitu explaining, reviewing dan restructuring dalam menyelesaikan lubang konstruksi yang terjadi pada siswa dalam menjawab soal mean, modus, dan median.

Kata Kunci: Defragmentasi, Lubang Konstruksi, Literasi Numerasi, Proses Berpikir

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INTRODUCTION

The average scores in the subjects of science, reading, and mathematics in 2022 dropped from those in 2018, reported to the OECD's (Organisation for Economic Co-operation and Development) PISA results for 2022. In Indonesia, only 18 percent of students completed the minimum competence level 2 requirements in mathematics, which is much below the OECD average of 69 percent of countries. Students are a minimum able to comprehend and recognise how easily problems can be represented mathematically without explicit teaching. According to the OECD average of nine percent, of a few Indonesian children succeed with outstanding results in mathematics, with a PISA score of a level 5 or 6. Students can model complicated circumstances methodically at this level, and they can choose, contrary, and analyze suitable approaches to solve problems in such scenarios (Harususilo, 2019), In contrast, approximately 27 percent of Indonesian students exhibited some level of reading competency in the 2018 PISA outcomes. While 71 percent of students fall short of the minimal proficiency in mathematics, 1b shows that students are limited to solving the simplest text comprehension problems. Stated differently, many Indonesian students still struggle when faced with circumstances that call for the application of mathematics to solve problems (Ilham, 2022).

The major issue of students' inability to solve mathematical problems throughout their learning needs to have a prompt remedy. Understanding concepts is a prerequisite for choosing problem-solving techniques, as students who have grasped the concepts will subsequently offer a variety of mathematical representations (Giawa, Gee, & Harefa, 2022). The study findings Zulkarnain and Budiman (2019), support the idea that students' capacity for problem-solving is influenced by their conceptual comprehension. The severe issue of pupils not understanding concepts when learning maths needs to have a remedy developed. Errors in the construction of mathematical concepts are one of the several causes of errors that happen. Students often duplicate the steps that teachers take them through when learning mathematics, therefore if the teacher provides additional exercises that differ from the ones that came before them or develop new ones, the students' answers won't match what the teacher expects. This affects kids' poorly built thought processes and knowledge. When conceptions are constructed incorrectly, mathematical concepts are not well understood, leading to knowledge that is not unified. Information that is improperly organized is known as fragmentation, and it frequently arises throughout the thinking process as a result of meaningless learning. This is particularly true when it comes to memorizing formulas and procedures (Wulandari, Usodo, Sutopo, Setiawan, Kurniawati, Kuswardi, & Aulia, 2020).

Problem-solving is a function of thinking, which is essential to human existence (Untung, Cahyono, & Sumarna, 2023). According to the findings of the researchers' observations during the course work in mathematics at SMP Negeri 6 Pasuruan, students continued to make mistakes when formulating ideas for numeracy literacy problems. Here, numeracy literacy is defined as the ability to use a variety of numbers and symbols to solve real-world problems, analyze data presented in the form of tables, charts, graphs, and diagrams, and then use the interpretation's findings to forecast and make decisions (Ilham, 2022). Many researchers continue to investigate the issue of numeracy literacy since many students, particularly in junior high school, are still unaccustomed to solving numeracy literacy problems.

According to research findings Logistica and Awalludin (2024), students classified as low ability are more likely to make mistakes at all procedural stages of analysis when it comes to answering numeracy literacy questions for class procedural process skills, including procedural writing answers related to solving numeracy literacy questions. The requirement for

unique abilities in solving numeracy literacy questions, such as frequently doing numeracy literacy exercises, and even self-confidence. Self-efficacy can be a source of inspiration, ideas, and even self-assurance, which can strongly inspire someone to appreciate the importance of numeracy literacy. Based on research by Malanua, Pomalato, and Damayanti (2024) which produces research results that students who have high self-efficacy can solve numeracy literacy questions well, both on the indicators of formulating, using, and interpreting the questions, on the other hand, students with low self-efficacy are only able to solve questions that students believe they can do well on the same indicators with easier level questions.

According to Putri and Awalludin (2024) research, students with high and low self-efficacy were able to reach indications of fluency, elaboration, and originality, whereas those with low self-efficacy were only able to obtain indicators of fluency. Students find it difficult to answer numeracy literacy questions, according to the findings of several earlier studies. When reading numeracy literacy questions, comprehending their meaning, converting the necessary information, and completing the steps involved in solving numeracy literacy questions, students with both high and low comprehension abilities need to pay particular attention to the areas where they make mistakes and aren't focused on the content of the question; instead, it concentrates on the steps to finish what is already known in the question and the procedural breakdown of the answer so that it is understandable to both the student answering the task and other people. Before addressing these problems, errors in comprehension of numeracy literacy questions are also crucial.

Students' inaccurate conception of the question's concept will have an impact on the question-solving process and outcome. Errors in idea formation might cause inaccuracies in students' comprehension. According to Subanji (2015), there are five types of faults in forming conceptions, namely: (1) pseudo-construction, namely the outcomes of students' mathematical constructs that deviate from what the pupils wrote, (2) hole creation, namely students, when building mathematical concepts, do not intact or imperfectly, (3) miss analogical construction, essentially the development of pupils' mathematical conceptions by analogies; yet, there are deviations in the use of analogical reasoning, (4) miss logical construction, or faults in logical thinking, in which the process of constructing concepts through logistical thinking is disrupted by deviations in the application of logical norms. Defragmentation of the thinking structure is one technique for reducing the potential of student

errors in concept construction. The process of defragmentation is disequilibrium and cognitive conflict, and scaffolding is used for restructuring students' thinking when they make errors in solving mathematical problems, enabling them to rectify their mistakes (Andriani, Triyanto, & Nurhasanah, 2021). Defragmenting students' thought processes is essential when building concepts and addressing numeracy literacy difficulties to lower student errors in answering questions overall.

According to Wahab, Buhaerah, Ahsan, and Busrah (2022), defragmentation refers to the response of restructuring students' thought processes to create coherent concepts. Scaffolding, construction process analysis, cognitive conflict, and disequilibrium are the stages involved in breaking down students' thought systems, and student error can reduce the use of defragmentation by scaffolding type Anghileri (2006) adopted three scaffolding indication steps: examining, explaining, and restructuring. Reviewing requires explaining and justifying pupils' actions and statements to ensure their understanding. Following that, explaining comprises the teacher demonstrating and narrating, while rebuilding entails rewording the task for the students, resolving semantic difficulties, streamlining the issue, and providing significance.

This explanation of the issue suggests that further research be done to determine the specific areas in which junior high school students make errors when attempting to answer numeracy literacy questions. This will allow researchers to deconstruct students' thought processes following the flaws discovered. Regarding the explanation above, the research equations are: (1) What is the hole construction process in students' thinking construction in solving numeracy literacy questions on statistical material? (2) What's the defragmentation process in solving the numeration literacy issue?

METHODS

This is a qualitative study with a descriptive approach, which focuses on explaining phenomena in detail, particularly the formulation of students' thinking when working on numeracy literacy tasks (Johnson & Christensen, 2019). The first stage will be to use tests, to discover different sorts of student errors in conceptual construction and mathematical problem-solving. Tests students that there are two questions related to numeracy literacy questions in statistical material, and then the students' answers will be adjusted to the indicators of Polya (2014) theory, namely (1) understanding the problem by

defining the unknown, defining the conditions of the problem; (2) outlining the plan, namely finding a solution strategy and completing it in stages; (3) implementing the plan, namely carrying out all calculation. Second, use a qualitative approach to the idea of hole formation to explain students' process errors in construct mathematics. There are three types of answers: completely correct, which includes the concept, the process, and the results of students' responses; actual but with construction holes, which means that the ideas built by students in solving the problem are incomplete, so the process is incorrect; and wrong from the overall answer, which includes the concept, the process, and the final result. Still, the solution is correct, and the student is considered to have a construction hole. The researcher carefully chose research respondents for in-depth interviews based on exam results and class instructor recommendations, employing a purposive sample technique that matched specified requirements (Creswell & Poth, 2016). The researcher will investigate and interview two students of thirty-two. The subject criteria are as follows: (1) students have learned about literation numeracy, (2) students can convey the outcomes of their thinking vocally and in writing, and (3) students encounter building errors in construction holes based on the result of the test. Three academics, two lecturers, and one teacher, reviewed the test and interview items and assessed their relevance, applicability, and intelligibility. The exam instrument includes two statistical material questions, mean, mode, and median, which are tailored to numeracy literacy questions. The researcher used Cronbach's Alpha to assess the reliability of the test answers, whereas the interviews were unstructured, with the list of questions remaining on a topic related to the construction of the concept of mean, median, and mode, but the direction of the interview evolved spontaneously based on the respondents' responses and interactions.

A group of junior high school students in Pasuruan City, specifically from SMP Negeri 6 Pasuruan, will be thoroughly examined and interviewed by researchers using the theory of the defragmentation of students' cognitive structures as a basis (Damayanti, Subanji, & Sukoriyanto, 2020; Wahab, Buhaerah, Ahsan, & Busrah, 2022). Thirty-two students from class VIII D at SMP Negeri 6 Pasuruan for the 2023–2024 academic year and conducted in throughout Mei 2024. Interviews and test questions for numeracy literacy are the research's instruments. Based on how effectively students respond to questions about numeracy and literacy that are part of the requirements to construct holes, the study subjects are split up. The students will then have in-

depth interviews about their responses (Subanji, 2016). This research procedure is divided into three stages (Sugiyono, 2016). In stage 1, the researcher creates research instruments, such as observation sheets, interview sheets, and test sheets for numeracy literacy. In stage 2, the researcher verifies the student work results, which are adjusted using Polya theory and regrouped based on the characteristics of the construction hole. Finally, stage 3 is the reporting stage, where the results are further analyzed and discussed in the results and discussion and conclusions. In the analysis process, the interview results use thematic analysis, which is the method of identifying, interpreting, and reporting patterns or themes in the data with stages. (1) familiarising with the data, (2) initial coding, (3) searching for themes, (4) reviewing themes, (5) identifying and naming themes, and (6) creating a report (Braun & Clarke, 2012). Data triangulation used in this research is source triangulation, namely comparing information obtained from respondents, namely students, with information from teachers.

RESULTS AND DISCUSSION

1. Results of Student Responses to Numeracy and Literacy Questions in Construction Holes

Students often struggle to distill general mathematical ideas into more focused ideas while building mathematical concepts. When pupils don't fully grasp concepts, this can occur. Although they think the response is right, students find it difficult to provide an explanation when questioned again during interviews, and even their responses frequently conclude their understanding of mathematics. Construction gaps occur when students are unable to assemble an idea as a whole (Subanji, 2015). Using the outcomes of the students' replies that were incorporated into the building hole thinking structure process, researchers screened the students' answers when working on numeracy literacy problems to conduct a defragmentation procedure. Out of the thirty-two students who responded to the numeracy literacy questions, eight fell into the construction hole category, five failed to respond, and nineteen students properly solved the questions, which focused primarily on statistics material. The following students are modeled while they work through numeracy literacy questions and fall into the construction hole category.

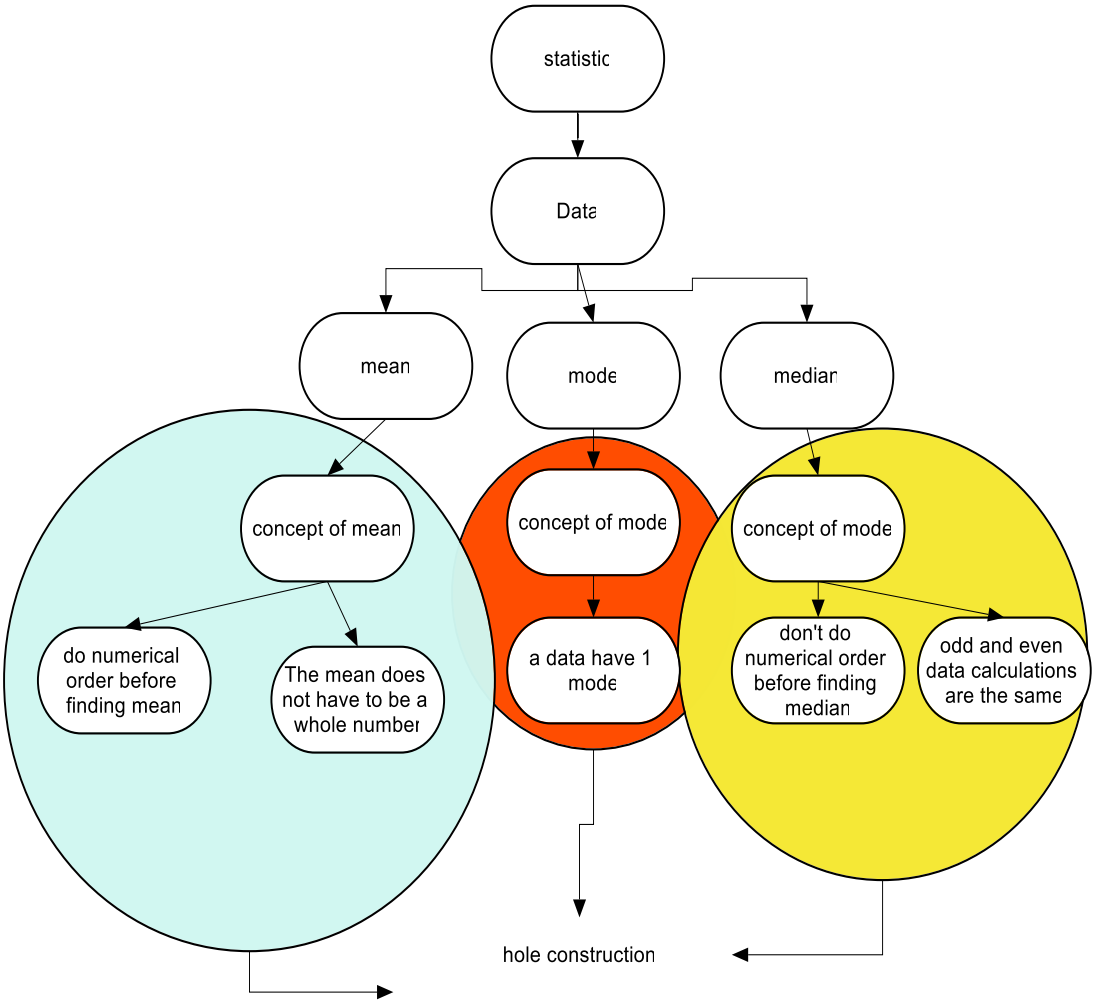

Figure 1. Hole Construction Process That is Often Encountered While Solving Statistics Exercise

Students encounter construction holes in numeracy literacy questions, particularly when statistics are presented, as figure 1 illustrates. Construction flaws occur when solving mean, mode, and median problems. It is described as follows.

Many students make blunders when developing concepts relating to mean, median, and mode, specifically:

a. Students still occasionally misunderstand the difference between mean and median, which is that sorting the data is necessary before determining the mean. In addition, since the mean of whole numbers is a concept they are familiar with, students typically round off their responses.
b. When asked to determine the mode value for a set of data based on its frequency, for instance when two huge data sets have the same frequency,

students typically respond that the data does not have a mode when asked about the idea of mode.

c. When it comes to the median notion, students are easily tricked by data that is both odd and even, particularly when there is a lot of it.

According to the National Numeracy Literacy Movement Team of the Ministry of Education and Culture, indicators for numeracy literacy questions based on numeracy literacy guidelines are as follows (Koesoema, Sutjipto, Setiawan, Hanifah, Miftahussururi, Nento, & Akbari, 2017):

Table 1. Numeracy Literacy Question Instrument Indicator Scores

No	Question Number	Indicator	The score for Each Indicator
1	1	Able to solve practical problems in a variety of everyday circumstances using a variety of numbers or symbols.	12.5
		Capable of evaluating data presented in a variety of formats (tables, charts, graphs, narratives, etc.)	10
		Able to evaluate and make inferences from the outcomes of analyses conducted in order to make predictions.	27.5
2	2	Able to solve practical problems in a variety of everyday circumstances using a variety of numbers or symbols.	12.5
		Capable of evaluating data presented in a variety of formats (tables, charts, graphs, narratives, etc.)	10
		Able to evaluate and make inferences from the outcomes of analyses conducted in order to make predictions.	27.5

The following are the outcomes of students' responses to the numeracy literacy questions they worked on.

Table 2. Outcomes of Students' Responses to Numeracy Literacy Tasks

No.	Student Name	Indicator 1	Indicator 2	Total
1	R1	15	42.5	57.5
2	R2	17.5	47.5	65
3	R3	0	0	0
4	R4	47.5	45	92.5
5	R5	42.5	42.5	95
6	R6	17.5	0	17.5
7	R7	35	20	55
8	R8	45	50	95
9	R9	22.5	15	37.5
10	R10	45	50	95
11	R11	7.5	7.5	15
12	R12	42.5	50	92.5
13	R13	47.5	50	97.5
14	R14	25	10	35
15	R15	22.5	47.5	70
16	R16	47.5	45	92.5
17	R17	42.5	40	82.5
18	R18	35	42.5	77.5
19	R19	15	30	45
20	R20	25	45	70
21	R21	15	35	50
22	R22	42.5	47.5	90
23	R23	42.5	40	82.5
24	R24	25	47.5	72.5
25	R25	0	0	0
26	R26	0	0	0
27	R27	25	20	45
28	R28	35	47.5	82.5
29	R29	22.5	47.5	70
30	R30	25	47.5	72.5
31	R31	42.5	50	92.5
32	R32	22.5	40	62.5

Table 3. Descriptive Statistics Results of Students' Numeracy Literacy Test Results on Statistics Material

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance	Kurtosis	
								Statistic	Std. Error
Indicator 1	32	75.0	0	75.0	30.078	16.504	272.373	.523	.809
Indicator 2	32	75.0	0	75.0	36.563	18.564	344.657	.141	.809
Total Score	32	97.5	0	97.5	62.734	30.469	928.371	-.315	.809
Valid N (listwise)	32								

Based on table 3, using descriptive statistics obtained from 32 students who worked on questions number 1 and 2, the minimum score was 0, the maximum score was 75, and the average score obtained by students working on question number 1 was 30.078, and the student's score was obtained in working on question number 2 was 36.563, with the standard deviation value of students' scores in working on question number 1 being 16.5037 and for question number 2 the standard deviation was 18.5649.

Table 4. Reliability Test Results of Student Test Instruments

Cronbach's Alpha	Cronbach's Alpha based on Standardized Items	N of Items
.844	.847	2

Table 4 shows that the results of the reliability test using Cronbach's Alpha are 0.844 ≥ 0.60, so it can be concluded that the numeracy literacy test instrument for statistical material is reliable.

The following are the findings of the questions that students worked on while completing their numeracy literacy assignments.

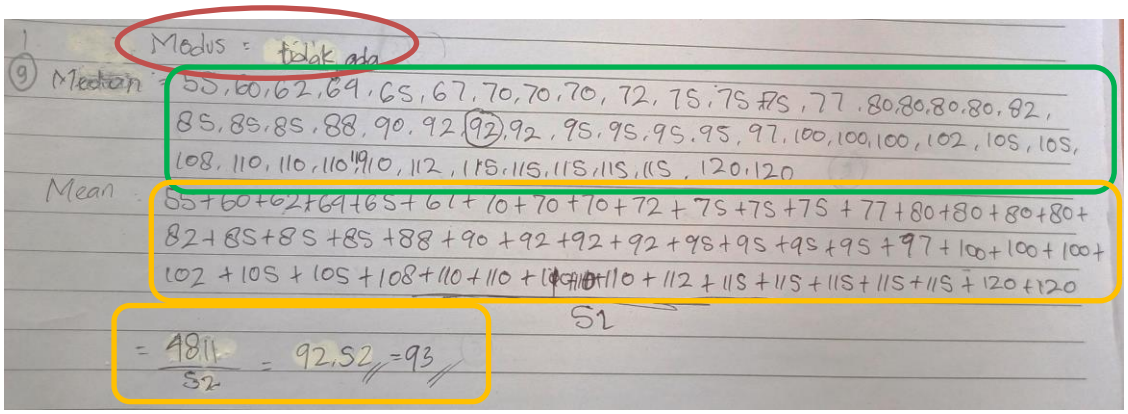

Figure 2. Students' Responses to Constructing Holes

Explanation:

= In response, students stated that the data did not contain a mode value.

= As they sort, students quickly circle the response that they believe to be the middle value to get the median.

= Students first sort the data to determine the mean value. They then round up the results.

Based on figure 2, it's found that thought processes are flawed and do not fully comprehend the notion of the mean when responding to question number 1. The notion of mean and median, according to the student's responses, is still flipped, meaning that sorting the data is necessary before determining the mean. Additionally, because they understand that the mean of whole numbers is the concept, students often round off their answers. Researchers performed in-depth interviews with respondents to get further information about the responses. The results are as follows:

"Construction of concepts formed in solving mean questions"

Researcher: *please attempt to provide a comprehensive explanation of your approach to answering question number 1.*

Respondent: *To solve the average problem, I sorted the numbers starting with the least and working my way up to the largest. Next, I determined how many of these numbers there were overall, and I divided that amount by the entire number of numbers.*

Researcher: *Why is it necessary to sort the statistics to determine the average value? Is there another method for figuring out the mean?*

Respondent: Yes, since you have to sort it first to find the average (D1).
For us to find the result after knowing the numerical sequence. Ma'am, if it's that way, it's just easier (D2). It is the only one I am aware of, ma'am. There isn't another one (D3).

The above-mentioned interviews with researchers and respondents resulted in code D1, in which students encountered construction holes connected to the idea of the median, specifically while answering media questions, students sorted the data from small to large. Based on interview codes D2 and D3, students believe in using this strategy because it is the method they are familiar with and they believe it is easier.

Researcher: Then, in order to make things easier, do you sort them before using the same procedure to obtain the mean, median, and mode? Explain
Respondent: Certainly, ma'am
Researcher: Do you utilize the method to determine the mean, median, and mode after
sorting the data in order? Are you certain that sorting the three answers is the first step in finding the answer?
Respondent: (quite a while of silence). Then I chuckled and said, "Ma'am, I'm not sure either." (D4)

Based on the findings of the researcher's interviews with participants, it is evident from the responses to the first question that students have a limited understanding of the idea of the mean. The cognitive frameworks of the students are still being developed. All phases must be arranged from smallest to largest number, but students' comprehension is restricted to the ideas of mean, mode, and median. When the researcher repeated the question to determine the students' confidence in their replies regarding sorting the data to find meaning, the researcher also reassured the students about the similar question for mode and median, which asked whether the data was also sorted. The student then reacts silently for a moment with code D4 before questioning his belief.

The following is provided for the subsequent interview concerning the questions on the mode in question number 2.

"Construction of concepts formed in solving median and mode questions"

Researcher: *please attempt to provide a comprehensive explanation of how you addressed Question number 2.*

Respondent: *first, I sort the data, ma'am, in order to get the mode from the maximum amount of data. I next sort the data again to find the median value, and last, I search for the middle value.*

Researcher: *To answer question number 2, how do you determine the mode when there are two values with the same frequency or quantity of data?*

Respondent: *There is no mode in the answer (E1), Ma'am*

Researcher: *Explain your reasoning, researcher, for selecting the "no mode" response.*

Respondent: *Are they lacking a mode because two of the items in that set have the same amount of data (E2)?*

Researcher: *Do you think that?*

Respondent: *kept quiet for a while. It appears that you are confident in my response, ma'am (students' reasons are less compelling). (E3)*

Based on the interview results, when the researcher asked about the concept of mode again, the respondent answered that there is no mode because there are two data with the same maximum quantitative data, the respondent had an incomplete understanding of the concept (construction hole), as shown in codes E1 and E2. Respondents also gave uncertain replies, namely code E3, when the researcher confirmed them again.

The following is provided for the subsequent interview concerning the questions on the median in question number 2.

"Construction of concepts formed in solving median questions"

Researcher: *in particular, if the data is even and the data is odd, what are the next procedures to determine the median after sorting it?*

Respondent: *Ma'am, I do not consider the statistics to be odd or even (F1). The left and right sides of the data are crossed out by hand using my method. (Researchers are given instructions by students on how to determine the median value).*

Researcher: *Do you know of any other methods for determining the median value?*

Respondent: *I don't know how, but yes, ma'am, we use odd and even data. But my approach is also the right one.*

Researcher: *Do you still employ your manual method if the data is large?*

Respondent: *Yes, ma'am, I now understand my approach better, but it will be (F2). Challenging for me if the data is big.*

According to the findings of the researcher's interviews with respondents, there is an incomplete construction of the median concept (construction holes), as evidenced by code F1, in which the respondent solved the median problem manually without considering whether the data was odd or even, and code F2, in which the respondent stated that the method he understands better, but when the data is large, he will have difficulties.

The graphic below shows fully the construction gaps associated with students' reasoning in solving mean, mode, and median questions before defragmenting.

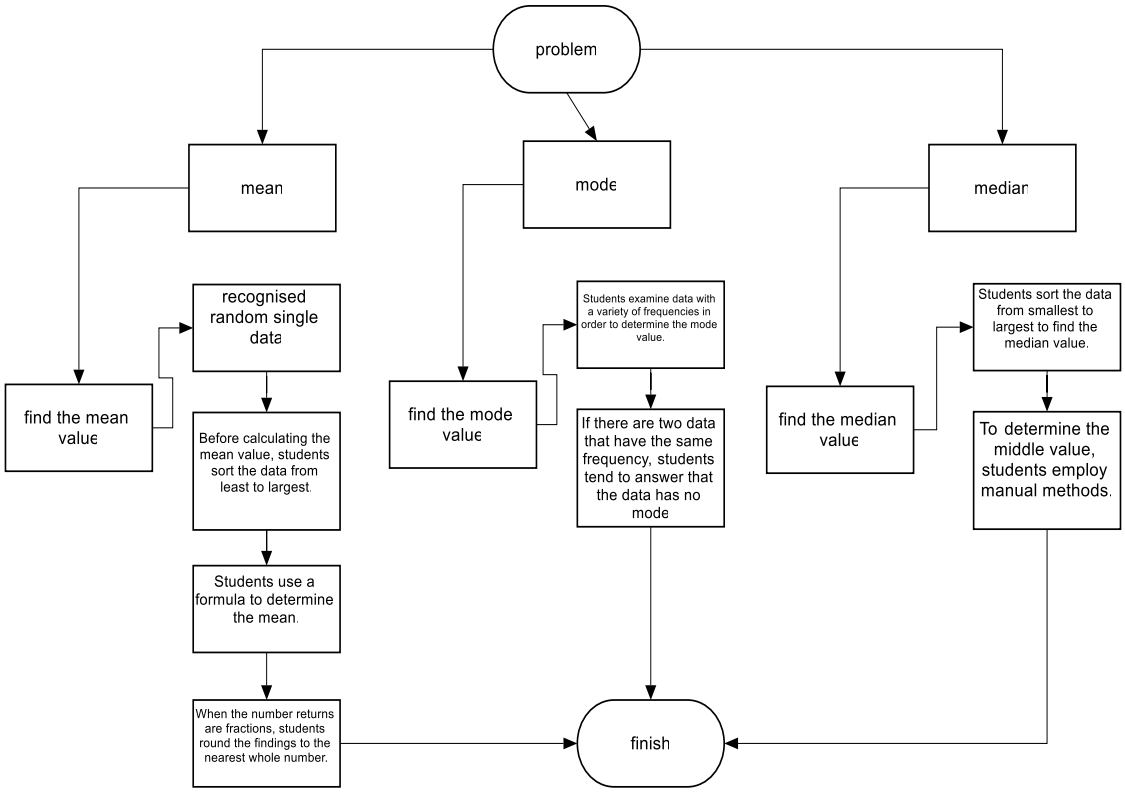

Figure 3. Before Defragmenting, Students' Thought Processes During Hole Formation in Mean, Mode, and Median Issue Solving

The picture in figure 3 shows how students' thought processes look when they answer questions about numeracy and literacy related to statistics. The building hole thinking structure of the students may be seen in this image as they are attempting to answer questions on the mean, mode, and median. The researcher used the defragmentation procedure to address the student's inadequate comprehension of the ideas of mean, mode, and median after determining the position of the student's construction hole thinking structure.

The researcher used defragmentation to comprehend the idea that students only partially understood. The defragmentation methods employed are disequilibration, conflict cognitive, and scaffolding, with type 2 scaffolding being used, according to (Khusnah, Ekawati, & Shodikin, 2023; Kurniati, Suhendra, Priatna, & Prabawanto, 2022), scaffolding level 2 (explaining, reviewing, and restructuring) is the defragmentation method used. In this level of scaffolding, teachers and students work closely together to improve students' understanding of incomplete thinking structures (Suci Wulandari, Hayati, & Hendriani, 2024).

Using this scaffolding, the teacher guides students through the explaining stage by posing questions that encourage them to comprehend the material thoroughly and simultaneously offer explanations. In the review phase, the teacher gives the students instructions to reconsider each answer and determine whether it is correct. Next, the teacher assists the students in concentrating their attention on the concept. In the restructuring phase, the teacher gives the students instructions to rectify any conceptual errors that have been communicated and offers a clear and accurate conclusion regarding concepts that the students do not understand (Puntambekar, 2022). Meanwhile, the researcher will create the following diagram following the defragmentation process.

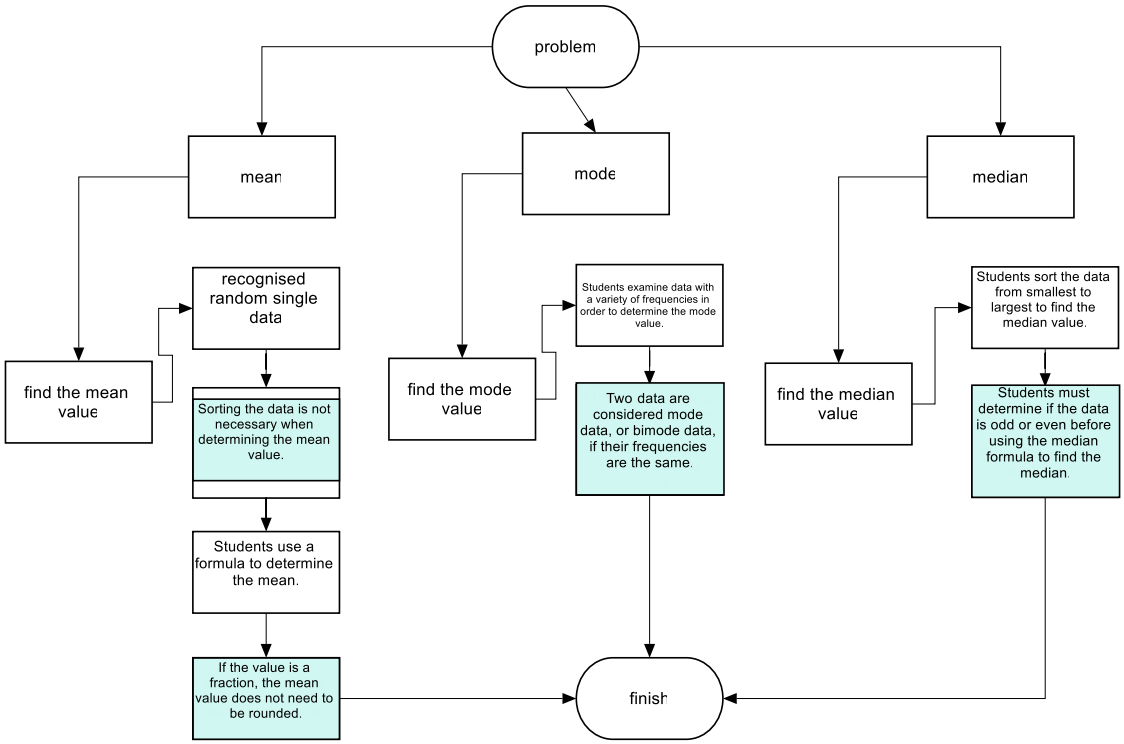

Figure 4. After Defragmenting, Students' Thought Processes

In figure 4, the researcher depicts the defragmentation process using a diagram, specifically the blue diagram, which shows that the researcher performed level 2 scaffolding-type defragmentation (explaining, reviewing, and restructuring) on the mean, mode, and median material. In figure 4, the researcher's defragmentation stage process at the scaffolding stage is demonstrated. Specifically, when students respond to the mean question, the researcher affirms the idea that they have been given, which is broken down in the mean section: to calculate the mean, the data does not need to be sorted; aside from that, the results There is no requirement to round based on the mean computation. To improve students' understanding of the notion, researchers also offer additional definitions of the mean content.

The researcher instructs students that data is split into two categories: odd data and even data, in order to deconstruct the idea of median. In the meantime, the data needs to be sorted from least to greatest in order to find the median. In the meanwhile, the researcher offers a thorough explanation of the mode notion, stating that it can have two values with an equal quantity of data. Researchers conducted more in-depth interviews with students to carry out the scaffolding steps, which are detailed below.

"Defragmentation of students' thinking structures in working on mean questions (solution of D1 until D4 code)"

Researcher: *What happens if the data is sorted and not sorted?* ***(disequilibrium)***
Respondent: *"I'm not sure, ma'am."*
Researcher: *attempt to figure out response number 1 in two different ways; the first is sorted (cognitive conflict),*
Are the mean values the same if your data isn't sorted in question number one? (Scaffolding-explaining) while the subsequent one doesn't seem to.
Respondent: *(number of students). It's the same, ma'am.*
Researcher: *what inferences can you draw from the two responses? (Scaffolding-Review)*
Respondent: *the mean value is the same when the means are sorted and unsorted.*
Researcher: *So, if you can already conclude, is it still appropriate to find the mean value? (Scaffolding-Restructuring)*
Respondent: *"No, ma'am." No need for me to sort*
Researcher: *Look up the meaning of mean and see if it can also be expressed as a*

decimal number. (Scaffolding-Explanation)

Respondent: *The respondent was silent and a little perplexed.*

Researcher: *Attempt to determine the mean value of these numbers if I have them (the instructor provides another example of data in the form of decimals). Examine the mean formula once more and attempt to determine the mean by reading it again. (Scaffolding-Review)*

Respondent: *"Yes, ma'am, you can calculate the mean."*

Researcher: *What conclusions can you draw?*

Respondent: *Fractional numbers are OK, ma'am.*

Researcher: *Is it implied that in the event of another mean inquiry, you will continue to round the fractional figures or leave them that way? (Scaffolding-Restructuring)*

Respondent: *no ma'am. No need to round*

"Defragmentation of students' thinking structures in working on mode questions (solution of E1 until E3 code)"

Researcher: *So, what happens if you solve the mode problem but find two pieces of data with the same maximum number or quantity? (disequilibrium) Why is there only one data in the data when there are multiples, such as five or ten? (disequilibrium)*

Respondent: *I am perplexed, ma'am.*

Researcher: *What about bi-mode and non-mode data? Try reading the text again, and instances of bi-mode and no-mode data (cognitive conflict)*

Respondent: *Yes, ma'am, there is data with two modes and data without modes. What does it mean to be a researcher? Try to explain (Scaffolding Explaining)*

Respondent: *For bi-mode, there are two data which are the mode values , ma'am. For those who don't have a mode, that is one data but a large number*

Researcher: *ok then what can you conclude is meaningful in answering question number? (Scaffolding-Reviewing)*

Respondent: *means the explanation is bi-mode ma'am, namely 110 and 115*

Researcher: *So, after there is a problem, look for another method, are you still confused? (Scaffolding-Restructuring)*

Respondent: *no ma'am, I understand now*

"Defragmentation of students' thinking structures in working on median questions (solution of F1 until F2 code)"

Researcher: *how can you calculate manually? What if the data is large? Do you still*

use the manual method to find the median? Isn't it difficult if you use the manual method (disequilibrium)

Respondent: yes ma'am, I will have difficulty if the data is large

Researcher: Then try calculating using the even or odd data formula based on the type of data whether it is odd or even and compare it with your manual method, (cognitive conflict). Is the result the same? (scaffolding-explaining)

Respondent: (students start counting and comparing). Yes, ma'am, same answer

Researcher: what can you explain about this process? (scaffolding-reviewing)

Respondent: to calculate median data, we look at whether the data is odd or even, then sort the data from smallest to largest then we use a formula according to the type of data whether it is odd or even

Researcher: Then, after there is a problem finding the median again, do you still use the manual method and your method earlier? (scaffolding-reviewing)

Respondent: no ma'am, I use what you said is easier

"The conclusion of the defragmentation process through interviews is as follows (code G)"

Researcher: what can you explain again regarding the mean, mode, and median so that you no longer make mistakes when working on the material mean mode median again? Try to conclude from the important points that we have discussed together

Respondent: to find the mean, the data does not need to be sorted, it can be calculated directly using the mean formula (code G1). To find the mode, look at the data that has the maximum quantity. If two data have the maximum quantity, the data still has a mode, namely those two data, but if all the data has the same quantity, then the data does not have a mode (code G2) and to determine the median value, you need to sort the data from smallest to largest then count the amount of data, whether the data is odd or even, then calculate the median value using the formula for odd data or even data so that not counting them one by one will take a lot of time (code G3).

Process of Thought Students who don't fully grasp a concept may have construction holes (Bahrudin, Indrawatiningsih, & Nazihah, 2019; Usodo, Aulia, Wulandari, Sutopo, Setiawan, Kurniawati, & Kuswardi, 2020;

Wulandari & Gusteti, 2021). Table 2 revealed that just 43.75% of students correctly answered question number 1, but 21 out of 32 students successfully answered question number 2. It is still evident that kids have trouble responding to questions about numeracy and literacy. Lack of conceptual grasp or poorly organized understanding of the material is the root cause of student difficulty.

Based on table 3, using descriptive statistics obtained from 32 students who worked on questions number 1 and 2, the minimum score was 0, the maximum score was 75, and the average score obtained by students working on question number 1 was 30.078, and the student's score was obtained in working on question number 2 was 36.563, with the standard deviation value of students' scores in working on question number 1 being 16.5037 and for question number 2 the standard deviation was 18.5649. According to the standard deviation results, the standard deviation value for question number 1 is less than that of question number 2, indicating that the data distribution for question number 1 is more homogeneous than question number 1.

Numerous questions in numeracy literacy are given in the form of narratives or even contextual questions, which can occasionally be challenging for students to answer. As a result, students frequently make mistakes when attempting to analyze the questions to provide an answer. Regarding numeracy literacy, there is a great deal of conceptual exploration as well as reasoning about the connections between mathematical ideas. This helps students learn how to solve problems appropriately, which comes from having a thorough understanding of the concepts (Pangesti, 2018). There are numerous reasons why students make mistakes when answering these questions: misconceptions about the questions themselves, internal reasons like being inattentive or unfocused, inadequate conceptual understanding, difficulties translating concepts into answers, and even mistakes made by the students themselves (Ulpa, Marifah, Maharani, & Ratnaningsih, 2021). Figure 2 makes it evident that students experience construction holes when attempting to solve the mean, median, and mode problems. The students' ability to categorise data which was only necessary when determining the median value was compromised after conducting in-depth interviews. Apart from that, students' thinking processes are also broken when it comes to the concepts of mean and mode, which can appear in two sets of data with the same frequency and when students round off the mean answer. Defragmentation is required, that is, a method for reconstructing students' thought processes so that the

ideas they are presented with can be fully organized (Aisya, Kusaeri, & Sutini, 2019). In the defragmentation process, there are stages such as cognitive conflict, scaffolding, and disequilibration (Cahyani, Kholid, Hamdani, & Asyhara, 2024). At the cognitive conflict stage, students are asked questions so that a conflict arises in the student's mind, the student will then think again about the answer he or she has previously given (Lesmana, Supratman, & Rahayu, 2022). Teachers can help students who answer with responses they are unsure about by providing scaffolding, step-by-step, so that students can explore subjects they may not be familiar with fully and learn how to solve issues effectively (Kholid, Sa'Dijah, Hidayanto, & Permadi, 2022). According to figure 4, the defragmentation process scheme is implemented through scaffolding stages. In other words, the researcher provides step-by-step guidance on the material means, mode, and median to respondents who encounter construction hole thinking structures, meaning that the concept they had received was incomplete.

The steps that the researcher outlines are similar to this: first, the researcher asks targeted questions and points them in the direction of questions that students in the construction hole category have answered. Next, the researcher conducts a review stage, in which students receive step-by-step assistance with alternative solutions that lead back to the correct concept so that the concepts that students understand can remain intact. Finally, the researcher strengthens the students' concepts that have fully developed from the in-depth interview process following the review stage. This is known as the restructuring stage, which is the stage of strengthening and forming the students' concept of understanding as a whole by drawing conclusions about what concepts have been built and understood by students. The teacher's assistance in the defragmentation process will have a significant impact on the student's overall thinking structure; hence, if the student gains a thorough comprehension of the topic, their thinking structure will also be good (Al-Samarraie, Teo, & Abbas, 2013). Teachers can carry out defragmentation in numerous ways other than just the scaffolding process; in the construction process analysis section, for example, by interpreting the problem at this stage therefore that the problem understood can be in the same direction (Prayitno, Purwanto, Subanji, Susiswo, & As'ari, 2020); in cognitive conflict, where researchers and students will encounter a conflict or gap regarding the point of view of specific information or knowledge (Bouzidi & Gendolla, 2023; Pratiwi, Nusantara, Susiswo, & Muksar, 2022); and disequilibrium, which is a stage for

providing information or reinforcement to help students experience a balance that allows them to connect one concept with another (Nizaruddin & Kusmaryono, 2023).

Based on the interview results, the researcher provides defragmentation of the construction holes experienced by students when working on mean questions, namely disequilibrium (asking questions that confuse students and thinking again to compare the answers), then the teacher provides reflection assistance, and finally cognitive conflict, namely assistance provided to correct the mistakes made happens to students by presenting diverse questions and examples so that they encounter conflict in their ideas and achieve the proper answer, and scaffolding, namely explaining, reviewing, and restructuring. The researcher reiterated what had been discussed with the respondents by conducting interviews to determine whether the results of the defragmentation process were well received.

The results obtained are through interview-based defragmentation, understanding students' thinking structures through discussions of the stages of disequilibrium, cognitive conflict, and scaffolding (reviewing, explaining, and restructuring) can be more easily understood directly by students. Based on the final results of the interviews in codes G1, G2, and G3, students can conclude that steps must be taken to prevent incomplete understanding of concepts or construction. This research is similar to research conducted by (Kurniati, Suhendra, Priatna, & Prabawanto, 2022), where scaffolding through the process of explaining, reviewing, and reviewing can resolve student errors in working on geometry such as: (1) errors in understanding concepts, (2) errors in operating settings, (3) errors in calculating, (4) misapplication of principles, (5) algorithm writing errors, (6) random response, and (7) errors in drawing. The research carried out by researchers is also in line with research by Andriani, Triyanto, and Nurhasanah (2021) where the defragmentation process was carried out using stages of disequilibrium, cognitive conflict, and scaffolding. Aside from defragmentation, many approaches to the problem of students' thinking structures are influenced by different teaching methodologies and student competencies. Different students' mathematical ability backgrounds influence students' success factors when they employ routine processes, but it appears that this does not define students' capacity to reason about an issue (Rocha & Babo, 2024).

CONCLUSION

Based on the results of research and discussion, it can be concluded that students' conceptual thinking errors in solving statistics problems are: (1) in the mean, students still sort data from smallest to largest when calculating the mean value, and students tend to round their answers to whole numbers; (2) in mode, students still do not understand properly about bi-mode data and data that does not have a mode; (3) in median, after students sort the data, students tend to calculate the median manually to determine the median. The second conclusion is that the form of defragmentation that can be carried out through a qualitative approach such as interviews, namely through a process of disequilibrium, cognitive conflict, and scaffolding (reviewing, explaining, and restructuring) can be more easily understood directly by students and the effectiveness of defragmentation can be seen through code G1, G2, and G3. The research's drawback is that it still employs defragmentation through an interview or qualitative method and cannot assess its effectiveness quantitatively. So, for future research, a combined method can be adopted, namely by testing the effectiveness of employing quantitative and qualitative defragmentation therefore if you take a quantitative approach, more data can be used.

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REFERENCES

Aisya, S., Kusaeri, K., & Sutini, S. (2019). Restrukturisasi berpikir siswa melalui pemunculan skema dalam menyelesaikan soal ujian nasional mata pelajaran matematika. *Jurnal Review Pembelajaran Matematika*, *4*(2), 157–165. https://doi.org/10.15642/jrpm.2019.4.2.157-165.

Al-Samarraie, H., Teo, T., & Abbas, M. (2013). Can structured representation enhance students' thinking skills for better understanding of E-learning content? *Computers and Education*, *69*, 463–473. https://doi.org/10.1016/j.compedu.2013.07.038.

Andriani, S. P., Triyanto, T., & Nurhasanah, F. (2021). Defragmentation thinking structure to overcome errors in addressing mathematical problem. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(1), 339–350. https://doi.org/10.24127/ajpm.v10i1.3441.

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, *9*, 33–52. https://doi.org/10.1007/s10857-006-9005-9.

Bahrudin, M. A., Indrawatiningsih, N., & Nazihah, Z. (2019). Defragmenting struktur berpikir siswa SMP dalam menyelesaikan masalah bangun datar. *IndoMath: Indonesia Mathematics Education*, *2*(2), 127–140. https://doi.org/10.30738/indomath.v2i2.4701.

Bouzidi, Y. S., & Gendolla, G. H. E. (2023). Is cognitive conflict really effortful? Conflict priming and shielding effects on cardiac response. *Psychophysiology*, *60*(2), e14169. https://doi.org/10.1111/psyp.14169.

Braun, V., & Clarke, V. (2012). *Thematic analysis.* American Psychological Association.

Cahyani, V. F., Kholid, M. N., Hamdani, Z., & Asyhara, S. A. (2024). Defragmentation of metacognition in solving ethnomathematics-oriented problems. *AIP Conference Proceedings*, *2926*(1). https://doi.org/10.1063/5.0182803.

Creswell, J., & Poth, C. (2016). *Qualitative inquiry and research design: Choosing among five approaches*.

Damayanti, P. A., Subanji, & Sukoriyanto, S. (2020). Defragmentasi struktur berpikir siswa impulsif dalam memecahkan masalah geometri. *Jurnal Pendidikan: Teori, Penelitian, Dan Pengembangan*, *5*(3). https://doi.org/10.17977/jptpp.v5i3.13239.

Giawa, L., Gee, E., & Harefa, D. (2022). Analisis kemampuan pemahaman konsep matematis siswa pada materi bentuk pangkat dan akar di kelas XI SMA negeri 1 Ulususua tahunpembelajaran 2021/2022. *AFORE: Jurnal Pendidikan Matematika*, *1*(1), 64–77. https://doi.org/10.57094/afore.v1i1. 437.

Harususilo, Y. E. (2019). Skor PISA 2018: Daftar peringkat kemampuan matematika, berapa rapor Indonesia? *Kompas.Com*.

Ilham, B. U. (2022). Harbuknas 2022: Literasi Indonesia peringkat ke-62 dari 70

negara.

Johnson, R. B., & Christensen, L. (2019). Educational research: Quantitative, qualitative, and mixed approaches 5th edition. In *USA: SAGE Publication*.

Kholid, M. N., Sa'Dijah, C., Hidayanto, E., & Permadi, H. (2022). Students' reflective thinking pattern changes and characteristics of problem solving. *Reflective Practice*, *23*(3), 319–341. https://doi.org/10.1080/14623943.2021.2025353.

Khusnah, K., Ekawati, R., & Shodikin, A. (2023). Student's error in solving change and relationship-PISA problem and its scaffolding. *Journal of Mathematical Pedagogy (JoMP)*, *4*(1), 9–20. https://doi.org/10.26740/jomp.v4n1.p9-20.

Koesoema, D. A., Sutjipto, S., Setiawan, D. I., Hanifah, N., Miftahussururi, M., Nento, M. N., & Akbari, Q. S. (2017). *Pedoman penilaian dan evaluasi: Gerakan literasi nasional*. Jakarta: Badan Penelitian dan Pengembangan dan Perbukuan.

Kurniati, N., Suhendra, S., Priatna, N., & Prabawanto, S. (2022). An exploration student's errors in solving trigonometric ratio problems with its scaffolding. *Aksioma: Jurnal Program Studi Pendidikan Matematika*, *11*(3), 2235–2247. https://doi.org/10.24127/ajpm.v11i3.5233.

Lesmana, B. N., Supratman, S., & Rahayu, D. V. (2022). Defragmentation of the false-pseudo thinking process of students in solving mathematical connection ability problems. *Mathline: Jurnal Matematika Dan Pendidikan Matematika*, *7*(2), 250–268. https://doi.org/10.31943/mathline.v7i2.284.

Logistica, R. S. A., & Awalludin, S. A. (2024). Analysis of students'errors in solving literacy and numeracy problems: A newman procedure approach. *MaPan: Jurnal Matematika Dan Pembelajaran*, *12*(1), 47–63. https://doi.org/10.24252/mapan.2024v12n1a4.

Malanua, M. K., Pomalato, S. W. D., & Damayanti, T. (2024). Kemampuan literasi matematika siswa dalam menyelesaikan soal cerita aljabar ditinjau dari self efficacy matematika. *FIBONACCI: Jurnal Pendidikan Matematika Dan Matematika*, *10*(1), 1–20.

Nizaruddin, & Kusmaryono, I. (2023). Transforming students' pseudo-thinking into real thinking in mathematical problem solving. *International Journal of Educational Methodology*, *9*(3). https://doi.org/

10.12973/ijem.9.3.477.

Pangesti, F. T. P. (2018). Menumbuhkembangkan literasi. *Indonesian Digital Journal of Mathematics and Education*, *5*(9), 566–575.

Polya, G. (2014). *How to solve it: A new aspect of mathematical method* (Vol. 34). Princeton university press.

Pratiwi, E., Nusantara, T., Susiswo, S., & Muksar, M. (2022). Routines' errors when solving mathematics problems cause cognitive conflict. *International Journal of Evaluation and Research in Education*, *11*(2), 773–779. https://doi.org/10.11591/ijere.v11i2.21911.

Prayitno, L. L., Purwanto, P., Subanji, S., Susiswo, S., & As'ari, A. R. (2020). Exploring student's representation process in solving ill-structured problems geometry. *Participatory Educational Research*, *7*(2), 183–202. https://doi.org/10.17275/PER.20.28.7.2.

Puntambekar, S. (2022). Distributed scaffolding: Scaffolding students in classroom environments. *Educational Psychology Review*, *34*(1), 451–472. https://doi.org/10.1007/s10648-021-09636-3.

Putri, R. A., & Awalludin, S. A. (2024). Analisis kemampuan berpikir kreatif matematis ditinjau dari self efficacy dalam menyelesaikan soal berbasis literasi dan numerasi. *FIBONACCI: Jurnal Pendidikan Matematika Dan Matematika*, *10*(1), 51–64.

Rocha, H., & Babo, A. (2024). Problem-solving and mathematical competence: A look to the relation during the study of Linear Programming. *Thinking Skills and Creativity*, *51*, 101461. https://doi.org/10.1016/j.tsc.2023.101461.

Subanji. (2015). *Teori kesalahan konstruksi konsep dan pemecahan masalah matematika*. Universitas Negeri Malang.

Subanji, S. (2016). *Teori defragmentasi struktur berpikir dalam mengosntruksi konsep dan pemecahan masalah matematika*. Universitas Negeri Malang.

Sugiyono. (2016). *Metode Penelitian Pendidikan: Pendekatan Kuantitatif, Kualitatif, dan R&D*. Bandung: Algesindo.

Ulpa, F., Marifah, S., Maharani, S. A., & Ratnaningsih, N. (2021). Analisis kesalahan siswa dalam menyelesaikan soal kontekstual pada materi bangun ruang sisi datar ditinjau dari teori nolting. *Square: Journal of*

Mathematics and Mathematics Education, *3*(2), 67–80. https://doi.org/10.21580/square.2021.3.2.8651.

Untung, Cahyono, E., & Sumarna, N. (2023). Analisis proses berpikir dalam pemecahan masalah matematika siswa berdasarkan langkah polya ditinjau dari self efficacy disma negeri 1 Kendari. *Jurnal Pembelajaran Berpikir Matematika*, *8*(1).

Usodo, B., Aulia, I. I., Wulandari, A. N., Sutopo, Setiawan, R., Kurniawati, I., & Kuswardi, Y. (2020). Fragmentation of thinking structure and its impact to students' algebraic concept construction and problem solving. *Journal of Physics: Conference Series*, *1567*(3), 032006. https://doi.org/10.1088/1742-6596/1567/3/032006.

Wahab, A., Buhaerah, Ahsan, M., & Busrah, Z. (2022). Defragmenting the thinking structure of problem solving through cognitive mapping based on polya theory on pisa problems. *Journal of Mathematics Learning Innovation (JMLI)*, *1*(1), 93–97. https://doi.org/10.35905/jmlipare.v1i1.3388.

Wulandari, A. N., Usodo, B., Sutopo, Setiawan, R., Kurniawati, I., Kuswardi, Y., & Aulia, I. I. (2020). Fragmentation of thinking structure: Concept construction and problem solving in geometry of junior high school students. *Journal of Physics: Conference Series*, *1567*(3), 032007. https://doi.org/10.1088/1742-6596/1567/3/032007.

Wulandari, S., & Gusteti, M. U. (2021). Defragmentation of Preservice teacher's thinking structures in solving higher order mathematics problem. *Journal of Physics: Conference Series*, *1940*(1), 012099. https://doi.org/10.1088/1742-6596/1940/1/012099.

Wulandari, Suci, Hayati, R., & Hendriani, M. (2024). Studi literatur - scaffolding dengan metode defragmenting struktur berpikir masalah hots. *Dharmas Education Journal (DE_Journal)*, *5*(1), 15–25. https://doi.org/10.56667/dejournal.v5i1.1196.

Zulkarnain, I., & Budiman, H. (2019). Pengaruh pemahaman konsep terhadap kemampuan pemecahan masalah matematika. *Research and Development Journal of Education*, *6*(1), 18–27. https://doi.org/10.30998/rdje.v6i1.4093.