

Analysis of Students' Procedural Fluency in Solving Polynomial Division Problems in Senior High School

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Abstract:

Procedural fluency is one of the core dimensions of mathematical proficiency essential for students' success in solving mathematical problems; however, students often rely on a single strategy, use inefficient procedures, and make computational errors. This study aimed to analyze Grade XI MTK-2 students' procedural fluency in solving polynomial division problems at a senior high school in Pontianak. Using a qualitative descriptive design, data were collected from 19 students through written tests and interviews. Data were analyzed through data reduction, data display, and conclusion drawing, while validity was established through technique triangulation. The analysis focused on the indicators of accuracy, efficiency, and flexibility. The findings showed that students generally demonstrated procedural fluency by applying appropriate procedures and using more than one solution method, particularly long division and Horner's method. Difficulties in accuracy persisted due to basic calculation errors and conceptual misunderstandings regarding polynomial structure and remainders. These findings indicate that procedural fluency in polynomial division depends not only on procedural execution but also on conceptual understanding, highlighting the importance of integrating conceptual and procedural learning and providing students with opportunities to learn and compare multiple solution methods in algebra instruction.

Abstrak:

Kelancaran prosedural merupakan salah satu dimensi utama kecakapan matematis yang penting bagi keberhasilan siswa dalam menyelesaikan masalah matematika. Namun, siswa masih sering menunjukkan keterbatasan, seperti bergantung pada satu strategi penyelesaian, menggunakan prosedur yang kurang efisien, serta melakukan kesalahan perhitungan. Penelitian ini bertujuan untuk menganalisis kelancaran prosedural siswa dalam menyelesaikan soal pembagian polinomial pada siswa kelas XI MTK-2 di salah satu sekolah menengah atas di Pontianak. Penelitian ini menggunakan desain deskriptif kualitatif dengan melibatkan 19 siswa. Data dikumpulkan melalui tes tertulis dan wawancara, kemudian dianalisis melalui tahap reduksi data, penyajian data, dan penarikan kesimpulan, sedangkan keabsahan data diperoleh melalui triangulasi teknik. Analisis dilakukan berdasarkan indikator akurasi, efisiensi, dan fleksibilitas. Hasil penelitian menunjukkan bahwa siswa secara umum

telah memiliki kelancaran prosedural yang ditunjukkan melalui kemampuan menerapkan prosedur yang sesuai serta menggunakan lebih dari satu metode penyelesaian, terutama metode pembagian bersusun dan metode Horner. Namun demikian, kesulitan pada aspek akurasi masih ditemukan akibat kesalahan perhitungan dasar dan miskonsepsi konseptual yang berkaitan dengan struktur polinomial dan sisa pembagian. Temuan ini menunjukkan bahwa kelancaran prosedural dalam pembagian polinomial tidak hanya bergantung pada pelaksanaan prosedur, tetapi juga pada pemahaman konseptual. Oleh karena itu, pembelajaran aljabar perlu mengintegrasikan pemahaman konseptual dan prosedural serta memberikan kesempatan kepada siswa untuk mempelajari dan membandingkan berbagai metode penyelesaian.

Keywords:

Mathematical Proficiency, Procedural Fluency, Polynomial Division Problems

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INTRODUCTION

Procedural fluency constitutes one of the core dimensions of mathematical proficiency that is essential for students' success in solving mathematical problems (Brahier, Leinwand, & Huinker, 2014; Cartwright, 2018; NCTM, 2000; National Council of Teachers of Mathematics, 2023). According to Kilpatrick, Swafford, and Findell (2001), procedural fluency refers to the ability to carry out mathematical procedures accurately, efficiently, and flexibly. Students with adequate procedural fluency are expected to apply appropriate procedures correctly, select efficient solution strategies, and use more than one method when solving mathematical problems. Such competence plays an important role in supporting conceptual understanding and facilitating the learning of more advanced mathematical topics (Coddington, Burns, & Lukito, 2011; Foster, 2016; Firdaus, 2019).

Despite its importance, numerous studies have reported that students' procedural fluency remains below the expected level. Procedural fluency is commonly assessed through three indicators: accuracy, efficiency, and flexibility (Newton, Record, & Mello, 2024; Bay-Williams & SanGiovanni, 2021; Kilpatrick, Swafford, & Findell, 2001; Russell, 2000). However, previous research revealed that many students were unable to solve problems using

multiple strategies, frequently performed inefficient procedures, and made repeated computational errors. For instance, Sari, Yusmin, and Nursangaji (2018) found that most students could not solve quadratic equation problems using at least two methods, failed to simplify solution procedures, and often produced inaccurate results. Similarly, Haryandika, Utami, and Prihatiningtyas (2017) reported that senior high school students demonstrated relatively low levels of procedural fluency. These findings indicate a discrepancy between the ideal expectations of procedural fluency and students' actual performance.

One mathematical topic that requires strong procedural fluency is algebra. Algebra serves as a foundation for learning many other mathematical topics, and difficulties in algebra often hinder students' overall mathematical achievement (Nasir, Hashim, Zabidi, Awang, & Zaihidee, 2013). Polynomials represent a central topic within algebra that requires mathematical reasoning and cannot be adequately addressed through memorization alone. A polynomial is an algebraic expression consisting of multiple terms, where the variables are raised only to non-negative integer powers (exponents) (Aksin, Miyanto, & Astuti, 2019). Within algebra, polynomials constitute an important topic that requires students to apply algebraic concepts and procedures appropriately. Solving polynomial problems involves manipulating algebraic expressions, applying operational rules, and carrying out a sequence of procedures accurately. Therefore, students need not only conceptual understanding but also strong procedural competence to solve polynomial problems successfully (Hidayah & Firdaus, 2025; Ekamornaroon, Ngiamsunthorn, Phaksunchai, & Chonchaiya, 2024; Sarirah & Nurmallasari, 2024; Wedyastuti, 2023).

Nevertheless, previous studies have shown that students continue to experience difficulties when solving polynomial related problems. Common difficulties include errors in algebraic operations, incorrect simplification processes, incomplete procedures, and inaccurate final answers (Huliman, Sandie, & Muchtadi, 2025; Ekamornaroon, Ngiamsunthorn, Phaksunchai, & Chonchaiya, 2024; Oktaviani, 2020). These findings are consistent with preliminary classroom observations, which revealed that students frequently made computational mistakes, incorrectly applied algebraic rules, and presented unsystematic solution procedures. Such conditions suggest that students' procedural fluency in polynomial topics requires further attention.

Several researchers have previously investigated procedural fluency with different focuses. For example, Pratidiana and Muhayatun (2021) analyzed

procedural fluency focusing on solving linear programming problems with the following indicators: (1) Students are able to apply procedures appropriately; (2) Students are able to select and utilize procedures; and (3) Students are able to modify procedures. Research by Safitri and Lestari (2022) analyzed junior high school students' procedural fluency in relation to self regulated learning, while Nurkhasanah and Ruli (2023) focused on students' procedural fluency in solving higher order thinking skills (HOTS) problems involving quadratic equations. More recently, Huliman, Sandie, and Muchtadi (2025) investigated students' procedural fluency in solving exponential problems. Although procedural fluency has been explored in various mathematical topics, studies focusing specifically on polynomial division remain limited. Therefore, this study addresses this gap by analyzing students' procedural fluency in solving polynomial division problems through the indicators of accuracy, efficiency, and flexibility.

Therefore, this study aims to describe students' procedural fluency in solving polynomial division problems based on indicators of accuracy, efficiency, and flexibility. This study contributes to the existing literature by providing a description of how students demonstrate procedural fluency across these three dimensions in the context of polynomial division, a topic that has received relatively little attention in previous research. The findings are expected to provide empirical evidence regarding students' strengths and difficulties in polynomial division and offer insights for mathematics teachers in designing learning strategies that encourage accurate, efficient, and flexible procedural performance.

METHODS

This study employed a descriptive qualitative approach to obtain an in-depth understanding of students' procedural fluency in solving polynomial division problems. The qualitative design was chosen because the study aimed to describe and interpret students' solution processes based on the indicators of procedural fluency, namely accuracy, efficiency, and flexibility. The study was conducted with 19 students from Grade XI MTK-2 at a senior high school in Pontianak.

The research instruments included a written essay test and a semi-structured interview guide. The test consisted of one polynomial division problem designed to assess students' procedural fluency based on three indicators: (1) accuracy, referring to the correctness of procedures and final

answers; (2) Efficiency, referring to the ability to use concise and appropriate solution procedures; and (3) Flexibility, referring to the ability to employ more than one solution method. Students were presented with a worked solution using the long division method, evaluated its correctness, identified any procedural errors, and then solved the problem using an alternative method. This task was intended to provide evidence of students' procedural fluency across the three indicators. Before data collection, the test and interview instruments were validated by experts in mathematics education to ensure their content validity and suitability for measuring procedural fluency.

Data were collected through measurement techniques using the written test and direct communication techniques through semi-structured interviews. Interview participants were selected purposively based on the results of the written test. The test item and interview protocol were reviewed by three experts in mathematics education to establish content validity. The validation process focused on content relevance, clarity of instructions, suitability for Grade XI students, and alignment with the procedural fluency indicators. Revisions were made based on the validators' suggestions before the instruments were administered.

The written test was administered to all 19 participants. Subsequently, three students were selected purposively for interviews. Three students were chosen because their written responses represented different characteristics of procedural fluency and demonstrated clear evidence related to the indicators of accuracy, efficiency, and flexibility. The selection was further supported by recommendations from the mathematics teacher regarding students' ability to communicate their thinking processes effectively during interviews.

Data analysis followed the interactive model proposed by Miles and Huberman (1994), which consists of data reduction, data display, and conclusion drawing. During the data reduction stage, students' written responses and interview transcripts were analyzed and categorized according to the procedural fluency indicators. The reduced data were then presented in the form of tables, figures, and narrative descriptions to facilitate interpretation. Finally, conclusions were drawn by identifying patterns of students' procedural fluency based on the indicators of accuracy, efficiency, and flexibility.

To ensure the trustworthiness of the data, technique triangulation was employed by comparing and cross checking information obtained from the written test and interview results. This procedure was conducted to verify the

consistency of the findings and strengthen the credibility of the data interpretation.

RESULTS AND DISCUSSION

Students' written responses and interview data were analyzed based on accuracy, efficiency, and flexibility in solving polynomial division problems using long division, Horner's method, and the method of undetermined coefficients. The distribution of students according to the solution methods used for each procedural fluency indicator is presented in figure 1.

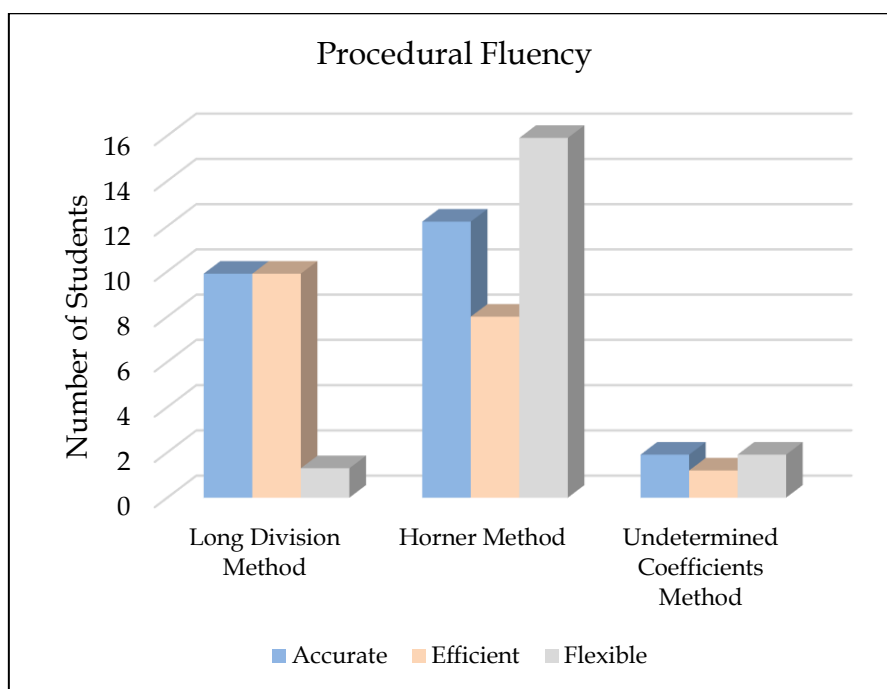


Figure 1. Procedural Fluency Data Based on Method of Solution

1. Accuracy

The accuracy aspect was analyzed by examining students' solution steps according to the methods used.

a. Solution Using the Long Division Method

Table 1 shows the distribution of students' accuracy based on the long division method.

Table 1. Distribution of Data on the Accuracy Using the Long Division Method

No	Accuracy Indicators for the Long Division Method	Frequency
1	Write the dividend and divisor polynomials correctly	19
2	Write the quotient correctly	12
3	Perform the calculation correctly	10
4	Write the remainder correctly	10
5	Identify the error in step 4 of the problem	12

Based on table 1, 100% of students correctly wrote the dividend and divisor in the division bracket, whereas only 53% successfully performed accurate calculations to obtain the correct quotient and remainder. The remaining students made errors during the division process, particularly when multiplying and subtracting polynomial terms. Some students incorrectly determined the coefficients or signs, which then affected subsequent steps and resulted in incorrect quotients and remainders. These errors indicate difficulties in accurately performing polynomial operations throughout the long division method.

b. Solution Using the Horner Method

Eighteen students applied Horner's method, whereas one student did not submit a written response. The distribution of accuracy indicators is presented in table 2.

Table 2. Distribution of Data on the Accuracy Using the Horner Method

No	Accuracy Indicators for the Horner Method	Frequency
1	Write the coefficients in the correct order	18
2	Write the value of k correctly	16
3	Perform the quotient correctly	13
4	Perform the calculation correctly	13
5	Write the remainder correctly	13

Based on table 2, 100% of students correctly arranged the coefficients, while 72% accurately obtained the quotient and remainder. The remaining students made errors during the synthetic division process, particularly when performing arithmetic operations involving coefficients. Some students incorrectly multiplied or added coefficients, resulting in inaccurate subsequent calculations and incorrect final answers. These errors indicate difficulty maintaining calculation accuracy throughout the procedure, which caused some students to stall in the solution process.

c. Solution Using the Undetermined Coefficients Method

There were two students who applied the method of undetermined coefficients. The complete results are presented in table 3 below.

Table 3. Distribution of Data on the Accuracy Using the Undetermined Coefficients

No	Accuracy Indicators for the Undetermined Coefficients Method	Frequency
1	Write down the examples $H(x)$ and $S(x)$ correctly in the correct order	2
2	Write down the form $P(x) = Q(x).H(x) + S(x)$ correctly	2
3	Determine the coefficient values correctly	2
4	Write the quotient correctly	2
5	Perform the calculation correctly	2
6	Write the remainder correctly	2

2. Efficiency

Data on the efficiency aspect of students' procedural fluency were analyzed by examining the appropriateness and conciseness of the solution procedures used by students.

a. Solution Using the Long Division Method

Table 4 shows the distribution of students' written solution steps when solving the polynomial division problem using the long division method.

Table 4. Distribution of Students' Written Steps in the Long Division Method

No	Solution Steps	Frequency	True	False
1	4 steps	1	0	1
2	7 steps	3	0	3
3	9 steps	1	0	1
4	10 steps	14	10	4

Based on table 4, more than half of the students who applied the long division method demonstrated efficiency by completing accurate solutions in 10 steps, whereas students who used fewer steps did not achieve accuracy due to incomplete solution procedures.

b. Solution Using the Horner's Method

Table 5 shows the distribution of students' written solution steps when solving the polynomial division problem using Horner's method.

Table 5. Distribution of Students' Written Steps in the Horner Method

No	Solution Steps	Frequency	True	False
1	5 steps	1	0	1
2	10 steps	9	9	0
3	11 steps	2	0	2
4	14 steps	4	3	1
5	17 steps	1	0	1
6	19 steps	1	1	0

Based on table 5, 50% of the students demonstrated efficiency in using Horner's method by completing accurate solutions in 10 steps, while longer procedures, despite being accurate, were considered less efficient because they still included repeated steps.

c. Solution Using the Undetermined Coefficients Method

Table 6 shows the distribution of students' written solution steps when solving the polynomial division problem using the method of undetermined coefficients.

Table 6. Distribution of Students' Written Steps in the Undetermined Coefficients Method

No	Solution Steps	Frequency	True	False
1	29 steps	1	1	0
2	33 steps	1	1	0

Based on table 6, it was found that students who used the method of undetermined coefficients required a longer sequence of steps.

3. Flexibility

The flexibility indicator in this study was operationalized as students' strategy repertoire, reflected in their ability to generate and apply more than one valid solution method for the same polynomial division problem. This operationalization focuses on the breadth of procedural knowledge rather than adaptive strategy selection across different problem contexts, as presented in table 7.

Table 7. Distribution of Data on Flexibility

Number of Methods Used	Frequency
1	1
2	16
3	2
Total	19

As shown in table 7, the majority of students (84%) demonstrated flexibility by using two solution methods, whereas 11% used three methods and only 5% solved the problem using one method.

The procedural fluency test showed variations in students' performance in terms of accuracy, efficiency, and flexibility, which were further examined through interviews with three selected students (KR, NAP, and YGIP).

Subject KR

Subject KR was able to solve the polynomial division problem using three different methods. The answer sheet for the subject KR in solving the polynomial division problem is presented below.

The figure displays three methods for solving polynomial division problems, with annotations identifying specific steps and errors.

Long Division Method:

- Identify the error in step 4 of the problem
- Write the quotient correctly
- Write the $P(x)$ dan $Q(x)$ correctly in the division brackets
- Write the remainder correctly

Using the long division method accurately and presenting 10 steps of the solution

Horner's Method:

- Write the value of k correctly
- Write the coefficients in the correct order

Using the Horner's method accurately and presenting 14 steps of the solution

Undetermined Coefficients Method:

- Write the quotient and remainder correctly

Figure 2. Subject KR Procedural Fluency Test Answers

Based on figure 2, it can be seen that subject KR was able to solve the problem using the long division method and Horner's method by writing the solution steps accurately, performing the calculations correctly, and obtaining the correct quotient and remainder; therefore, the solution was considered accurate. Furthermore, subject KR solved the problem using the method of undetermined coefficients, as shown in figure 3. The results obtained were consistent with those derived from the previous methods.

The image shows handwritten mathematical work for two methods of polynomial division. The top section shows the undetermined coefficients method for dividing $H(x) = ax^2 + bx + c$ by $S(x) = dx + e$. The student sets $P(x) = Q(x) \cdot H(x) + S(x)$ and expands it to $4ax^4 + (4b+4a)x^3 + (4c+4a-3a)x^2 + \dots$. They then solve for coefficients: $4b+4a=0 \Rightarrow b=-4$, $4c+4a-3a=-10 \Rightarrow c=-10+7 \Rightarrow c=-3$. The bottom section shows the long division method, where $4x^4$

Figure 3. Subject KR Procedural Fluency Test Answers

Based on the students' written responses, subject KR was able to apply the procedures accurately in all solution methods used. Subject KR obtained consistent final answers across each method applied. This finding is supported by the following interview excerpt:

- P : What methods did you use to solve the problem?
- KR : Long division, Horner's method, and the method of undetermined coefficients all produced the same result.
- P : Are you confident in your final answer? How did you ensure its correctness?
- KR : Yes, I am confident, because I reviewed each step of the solution presented in the problem and identified an error in one of the steps. After recalculating, I found that the result I obtained still led to the same final answer.

Based on the results of the written test and the interview, it is evident that subject KR did not make any errors in solving the problem using the long division method, Horner's method, or the method of undetermined coefficients; therefore, the solutions can be categorized as accurate and flexible.

Based on the test results, subject KR solved the problem using the long division method in 10 steps and Horner's method in 10 steps, and applied the

method of undetermined coefficients in 29 steps, all with accurate results. This is supported by the following interview excerpt:

- P : *Are there any parts of your solution that you think could actually be skipped or shortened without changing the result?*
- KR : *For the long division method, there are none. For Horner's method, I think there are; when calculating the remainder, it can be shortened by directly computing $\frac{17}{4} - (-3)\left(\frac{1}{2}\right) = \frac{23}{4}$. Meanwhile, the method of undetermined coefficients takes longer, although some steps in finding the coefficients can be simplified.*

KR subjects' responses indicated that the long division method was considered sufficiently concise for quadratic divisors and therefore did not require further simplification, as the quotient and remainder could be obtained directly through a systematic procedure. In Horner's method, students' solutions were already organized and systematically structured. However, they recognized that several intermediate calculations could be simplified without altering the underlying procedure. Specifically, students noted that certain arithmetic operations involved in determining the quotient and remainder could be performed mentally, thereby reducing the need to record every computational step explicitly. Consequently, the perceived efficiency of Horner's method was not attributed to a change in the algorithm itself, but rather to a reduction in unnecessary written calculations, resulting in a more concise and streamlined solution process. In contrast, students perceived the method of undetermined coefficients as relatively more time-consuming and cumbersome. This perception stemmed from the requirement to first construct an algebraic identity, then determine the values of the unknown coefficients through additional calculations before the quotient can be obtained. These extra stages increase the amount of symbolic manipulation required, leading students to regard the method as longer and less direct compared to Horner's method.

Based on the results of the written test and the interview, subject KR used fewer steps than the other students while maintaining accuracy; therefore, subject KR's solution can be considered efficient.

Subject NAP

Subject NAP solved the polynomial division problem using two methods, namely the long division and Horner's methods, as shown in figure 4.

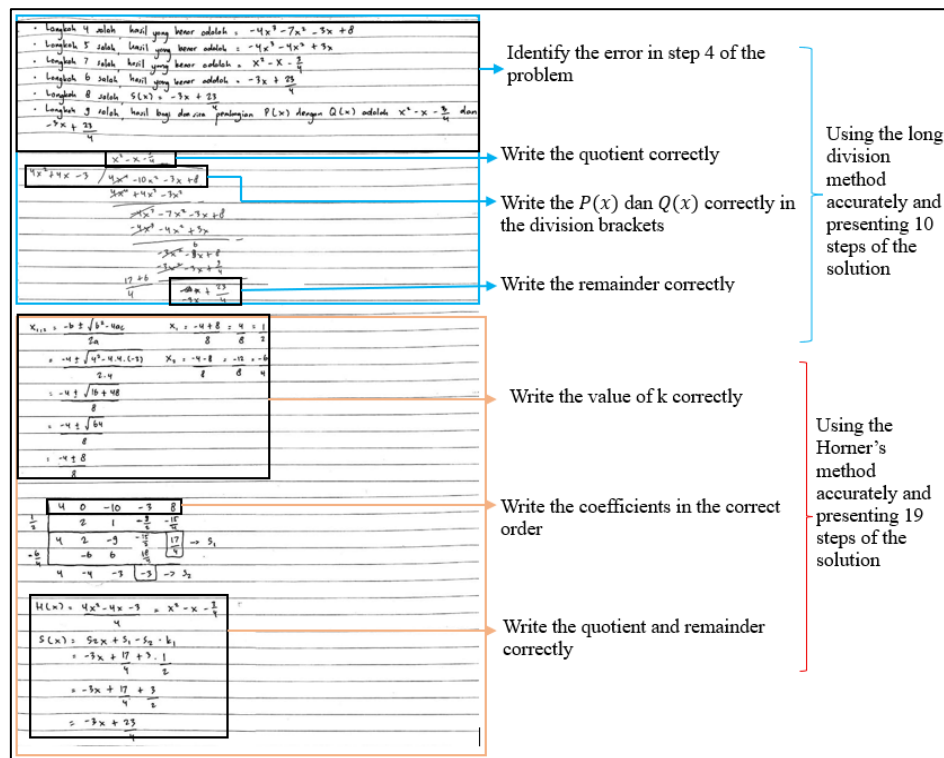


Figure 4. Subject NAP Procedural Fluency Test Answers

Based on figure 4, it can be seen that subject NAP was able to apply the procedures accurately for each method used. This finding is supported by the following interview excerpt:

- P : Are you confident in your final answer? How did you ensure its correctness?
 NAP : I am confident because I checked my work more than once and tried another method, and the answers were the same.

Based on the written test and interview results, subject NAP solved the problem accurately using the long division method without making errors and also demonstrated flexibility by applying an alternative solution method, namely Horner's method, as shown in figure 4.

Subject NAP obtained accurate results using the long division method in 10 steps and Horner's method in 19 steps. In applying Horner's method, NAP first determined the value of (k) by solving a quadratic equation, which contributed to the relatively lengthy solution process. During the interview, NAP acknowledged that some calculations could be shortened:

- P : Are there any parts of your solution that you think could be skipped or shortened without changing the result?

NAP : *I think the steps can be shorter. After substituting into the quadratic formula, we can immediately see that the value under the square root is 64, which gives 8.*

This response indicates that NAP recognized the opportunity to reduce unnecessary written calculations by mentally evaluating. However, in the written solution, NAP still recorded each intermediate step in detail. Therefore, while demonstrating awareness of the more concise approach, the overall solution remained relatively lengthy and was not categorized as efficient.

Subject YGIP

Subject YGIP solved the polynomial division problem using two methods, namely the long division method and Horner's method, as illustrated in figure 5.

1. Pembagian Horner

Langkah 2 salah: $x^2 + x$

Pembelian: $4x^2 + x - 5 / 4x^2 - 0x - 5 \times 10$

Langkah 4 salah: Pembelian: $-4x^2 + 0 - 7x^2 - 5x + 8$

Langkah 5 salah: Pembelian: $-4x^2 - 4x^2 + 5x$

Langkah 6 salah: Pembelian: $-3x^2 - 6x + 8$

Langkah 8 salah: Pembelian: $-3x^2 - 6x + 8$

Langkah 9 salah: Hasil bagi: $x + x$

Sisa: $-3x^2 - 6x + 8$

2. Pembagian Horner

$P(x) = 4x^3 - 10x^2 - 3x + 8$
 $Q(x) = 4x^2 + x - 5$

Langkah 1 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 2 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 3 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 4 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 5 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 6 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 7 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 8 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 9 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 10 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 11 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 12 salah: $4x^3 - 10x^2 - 3x + 8$

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Langkah 76 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 77 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 78 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 79 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 80 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 81 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 82 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 83 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 84 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 85 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 86 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 87 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 88 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 89 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 90 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 91 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 92 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 93 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 94 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 95 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 96 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 97 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 98 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 99 salah: $4x^3 - 10x^2 - 3x + 8$

Langkah 100 salah: $4x^3 - 10x^2 - 3x + 8$

Incorrectly identifying the steps

Making calculation errors

Write the $P(x)$ and $Q(x)$ correctly in the division brackets

Incorrectly writing the quotient and remainder

Using the long division method inaccurately and presenting 7 steps of the solution

Incorrectly writing the value of k

Write the coefficients in the correct order</

Figure 5. Subject YGIP Procedural Fluency Test Answers

Figure 5 indicates that subject YGIP demonstrated procedural flexibility by applying two different solution methods, which was further supported by the interview results:

P : *What method did you use to solve the problem besides long division?*

YGIP : *Horner's method.*

In solving the problem using the long division method, subject YGIP was able to correctly write $P(x)$ and the divisor $Q(x)$ in the division bracket and apply arithmetic operations appropriately. However, subject YGIP stopped at the step where $-3x^2 - 6x + 8$ should have been divided by the divisor. This finding is supported by the following interview excerpt:

P : *Do you understand the concept of the remainder in division?*

YGIP : *I understand that the degree of the remainder has to be lower than the degree of the divisor, so it can't be divided anymore. But I had trouble at that step because the result wasn't an integer, so I didn't continue.*

Based on the written test, subject YGIP also made a computational error when dividing $-4x^3$ by $4x^2$, which was written as x . This is supported by the following interview excerpt:

P : *Please review this division step. Do you think your calculation was correct?*

YGIP : *It seems there was a sign error; it should have been negative, but I wrote it as positive.*

Subject YGIP failed to correctly identify the appropriate solution steps, as evidenced by the misconception regarding the absence of an x^3 term. Based on the written test and interview results, errors occurred in the application of the long division method; therefore, the solution cannot be categorized as accurate.

Errors in determining the values of the k occurred in subject YGIP's use of Horner's method due to the omission of division by the coefficient of x^2 , as indicated by the interview results.

P : *What difficulties did you experience when using Horner's method?*

YGIP : *I obtained the values of k as 6 and -2, but they were very different from the results in long division. I tried recalculating, but it became more confusing.*

Errors in the solution process resulted in inaccurate quotients and remainders; therefore, despite completing the procedures in relatively few steps, subject YGIP's work cannot be considered efficient.

The findings indicate that accuracy was the most challenging aspect of procedural fluency for students, supporting the argument of Andal & Andrade (2022) that accurate execution of procedures often represents the greatest

difficulty in demonstrating procedural fluency. Although many students were able to recall and initiate the correct division procedures, their solutions frequently contained errors that prevented them from obtaining correct final answers. These findings suggest that procedural fluency involves more than the ability to remember algorithms; it also requires the accurate execution of each procedural step (Schulz, 2024; Bay-Williams & SanGiovanni, 2021; Kilpatrick, Swafford, & Findell, 2001).

The errors identified in this study can be categorized into two types. The first consisted of procedural errors, such as mistakes in sign operations, arithmetic calculations, and algebraic manipulations. Similar difficulties have been reported in previous studies, which found that students often lose accuracy when performing sequential operations involving positive and negative terms or when manipulating algebraic expressions (Huliman, Sandie, & Muchtadi, 2025; Oktaviani, 2020; Samsuddin & Retnawati, 2019; Sari, Yusmin, & Nursangaji, 2018).

However, the interview data revealed that some inaccuracies originated from deeper conceptual misunderstandings. Subject YGIP, for example, terminated the division process when encountering a remainder with fractional coefficients and failed to recognize the missing term in the dividend. This finding suggests that the student did not fully understand the fundamental structure of polynomial division. In polynomial division, missing terms must be represented by coefficients of zero to preserve the place value structure of the polynomial, and the division process continues until the degree of the remainder is less than the degree of the divisor. The student's decision to stop the procedure prematurely indicates a misconception about the meaning of the remainder and the completion criterion of polynomial division. Such findings support the view that procedural errors are often rooted in conceptual weaknesses rather than mere carelessness (Rohimah, Putri, & Helsa, 2026; Putri, Fua, & Halistin, 2025; Lenz, Reinhold, & Wittmann, 2024; Barham, 2020).

Some students demonstrated efficiency by applying the correct procedure concisely, without unnecessary repetition or excessive calculations (Huliman, Sandie, & Muchtadi, 2025). Conversely, some students produced lengthy solutions because they repeated calculations or noted intermediate steps that could have been simplified. Although these students were sometimes able to obtain the correct answer, their procedures were less efficient because the solution process took longer than necessary (Nurkhasanah & Ruli, 2023; Safitri & Lestari, 2022; Sari, Yusmin, & Nursangaji, 2018). The results also

revealed that inefficiency was not always caused by lengthy procedures. In some cases, it stemmed from students' incomplete understanding of the concepts and procedures involved in dividing polynomials (Nurkhasanah & Ruli, 2023). Students were unable to complete the division process and made errors when determining the quotient and remainder. These difficulties indicate that students did not fully understand the procedures required to solve the problem, making it difficult to identify a precise and concise solution path. This finding is consistent with previous research showing that limited conceptual understanding can hinder students' ability to select and implement efficient procedures.

The flexibility findings demonstrate that most students were capable of employing more than one method to solve the same problem (Bay-Williams & SanGiovanni, 2021; Sari, Yusmin, & Nursangaji, 2018; Russell, 2000). Interview results showed that students frequently used a second method to verify answers obtained through the first method, indicating an awareness that mathematical procedures are interconnected and can produce equivalent results. This behavior suggests that students possessed not only procedural knowledge but also an understanding of relationships among different algorithms. Such understanding is important because flexible strategy use has been associated with stronger mathematical reasoning and deeper procedural knowledge (Yusof, 2021).

While most students demonstrated the ability to apply multiple procedures, their performance was still constrained by computational errors and conceptual misunderstandings related to the structure of polynomial division. Therefore, instructional practices should emphasize not only procedural practice but also conceptual understanding of polynomial structure, the meaning of remainders, and the relationships among different division methods. Strengthening these conceptual foundations may help students perform procedures more accurately while maintaining efficiency and flexibility.

CONCLUSION

Based on the findings, students of Grade XI Mtk-2 generally demonstrated procedural fluency in solving polynomial division problems. The strongest aspect of their procedural fluency was flexibility. Students were able to apply the correct procedure and use more than one solution method, albeit to a limited extent. However, difficulties with accuracy remained apparent. The

findings suggest that these difficulties were caused not only by calculation errors but also by conceptual misunderstandings related to polynomial structure and the interpretation of remainders.

These findings imply that polynomial division instruction should integrate conceptual understanding with procedural practice and encourage students to compare different solution methods. Future research may investigate instructional strategies that support procedural fluency, examine the influence of students' foundational mathematical abilities, and explore procedural fluency in other mathematical topics.

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