

Mathematical Communication Construction Across Critical Thinking Levels in Function Problems Among High School Students

Nur 'Afifah^{1*}, Izwita Dewi², KMA Fauzi³

¹Universitas Muhammadiyah Sumatera Utara, Indonesia

^{2,3}Universitas Negeri Medan, Indonesia

¹Jl. Kapten Muchtar Basri, Medan, Indonesia

^{2,3}Jl. William Iskandar Ps. V, Deli Serdang, Indonesia

Correspondence Email: nurafifah@umsu.ac.id

Received April 20, 2026; Revised June 13, 2026; Accepted June 17, 2026

Available Online June 29, 2026

Abstract:

This study aims to describe the construction of mathematical communication in function problems among high school students with different levels of critical thinking. This study used a descriptive qualitative approach involving 32 students in grades XI-2 of Muhammadiyah 1 Medan Private High School. Data were collected through mathematical communication skills tests and critical thinking level instruments based on the Paul-Elder framework, as well as semi structured interviews. Data analysis was carried out by following the Miles, Huberman, and Saldana model, which included data reduction, data presentation, and conclusions drawn. The results showed that students with high levels of critical thinking had comprehensive mathematical communication skills, including the ability to consistently represent mathematical ideas in the form of writing, pictures, and symbols. Students with a critical thinking level are showing adequate communication skills, but still have difficulty with symbolic representation. Students with low critical thinking levels show very limited communication skills and have difficulty conveying mathematical ideas coherently. These findings suggest that students with different levels of critical thinking demonstrate different characteristics in constructing mathematical communication skills. The implications of this study emphasize the importance of designing learning strategies that simultaneously develop critical thinking skills and mathematical communication in mathematics learning in high school.

Abstrak:

Penelitian ini bertujuan untuk mendeskripsikan konstruksi komunikasi matematis dalam menyelesaikan masalah fungsi pada siswa SMA dengan tingkat berpikir kritis yang berbeda. Penelitian ini menggunakan pendekatan kualitatif deskriptif yang melibatkan 32 siswa kelas XI-2 SMA Swasta Muhammadiyah 1 Medan. Data dikumpulkan melalui tes keterampilan komunikasi matematika, instrumen tingkat berpikir kritis berdasarkan kerangka Paul Elder, serta wawancara semi-terstruktur. Analisis data dilakukan dengan mengikuti model Miles, Huberman, dan Saldana, yang meliputi reduksi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa siswa dengan tingkat berpikir kritis tinggi memiliki keterampilan komunikasi matematika yang komprehensif, termasuk kemampuan untuk secara

konsisten merepresentasikan ide matematika dalam bentuk tulisan, gambar, dan simbol. Siswa dengan tingkat berpikir kritis sedang menunjukkan keterampilan komunikasi yang memadai tetapi masih mengalami kesulitan dalam representasi simbolik. Siswa dengan tingkat berpikir kritis rendah menunjukkan keterampilan komunikasi yang sangat terbatas dan mengalami kesulitan menyampaikan ide matematika secara koheren. Temuan ini menunjukkan bahwa siswa dengan tingkat pemikiran kritis yang berbeda menunjukkan karakteristik yang berbeda dalam membangun keterampilan komunikasi matematika. Implikasi dari penelitian ini menekankan pentingnya merancang strategi pembelajaran yang secara simultan mengembangkan keterampilan berpikir kritis dan komunikasi matematis dalam pembelajaran matematika di sekolah menengah atas.

Keywords:

Mathematical Communication, Critical Thinking Level, Function, Senior High School

How to Cite: 'Afifah, N., Dewi, I., & Fauzi, K. M. A. (2026). Mathematical Communication Construction Across Critical Thinking Levels in Function Problems Among High School Students. *MaPan: Jurnal Matematika dan Pembelajaran*, 14(1), 122-132. <https://doi.org/10.24252/mapan.2026v14n1a7>.

INTRODUCTION

Mathematical communication skills are one of the fundamental competencies that students must master in learning mathematics. In the context of 21st century education, this ability is not just a mirror of numeracy skills, but rather a major prerequisite for students to be able to meaningfully express, discuss, and validate their mathematical understanding (Herawaty, Warni, & Angraini, 2021). The National Council of Teachers of Mathematics (NCTM) has established mathematical communication as one of the five standards of the ideal mathematics learning process, which requires students to be able to organize, consolidate, and analyze their mathematical thinking coherently (Afgani & Sutawijaya, 2022).

Reality in the field shows that the mathematical communication skills of high school students in Indonesia are still in the low to medium category (Hidayat & Husnussalam, 2023; Nurhayati & Zanthi, 2020). The 2022 Programme for International Student Assessment (PISA) data shows that Indonesia is ranked 68th out of 81 countries in the dimension of mathematical literacy, which includes mathematical communication skills (Ministry of Education and Culture, 2023). Functional material, as one of the staple materials in the high school curriculum, provides a rich mathematical context to evaluate

these abilities. However, students often experience profound misconceptions in function materials, especially related to notation, graph translation, and mapping relationships (Putri & Wijaya, 2021). Similar things are also found in the use of technology based learning media such as GeoGebra, which, when integrated with problem based learning models and ethnomathematics, has been proven to improve students' mathematical communication skills (Nasution, Irvan, & Coal, 2020).

One of the variables that is suspected to affect the construction of mathematical communication skills is the level of critical thinking of students (Firdaus, Wahyudin, & Herman, 2021; Rahmatiya & Miatun, 2020). Critical thinking in mathematics refers to an individual's ability to analyze arguments, evaluate evidence, and draw logical conclusions based on valid reasoning (Paul & Elder, 2020). Theoretically, critical thinking and mathematical communication skills are closely related cognitive-discursive skills. Sfard (2008) states that thinking is a form of communication with oneself (intrapersonal communication). Therefore, high level critical thinking serves as a cognitive regulator or metacognitive machine: this ability allows students to monitor, evaluate, and structurally improve the way they formulate mathematical equations and arguments before communicating them externally (interpersonal communication) (Yildiz & Baltaci, 2021).

Previous studies have examined these two variables more separately. Wijaya, Van den Heuvel-Panhuizen, and Doorman (2022), for example, examined critical thinking on contextual tasks without assessing mathematical communication, while Hidayat and Husnussalam (2023) analyzed the mathematical communication of high school students only in the context of statistical problems. Rahmatiya and Miatun (2020) examined mathematical problem-solving skills from a critical thinking perspective, but focused on general scores and did not touch on the microstructural construction of communication indicators. These studies are still descriptive and leave research gaps, namely, how different levels of critical thinking systematically form errors, language register selection, and structural constraints in the process of internal to external translation of the concept of function. This study seeks to fill this gap by using the Paul Elder framework to analyze how certain critical thinking profiles shape mathematical discourse on three main communication channels, namely written text, images, and mathematical expressions.

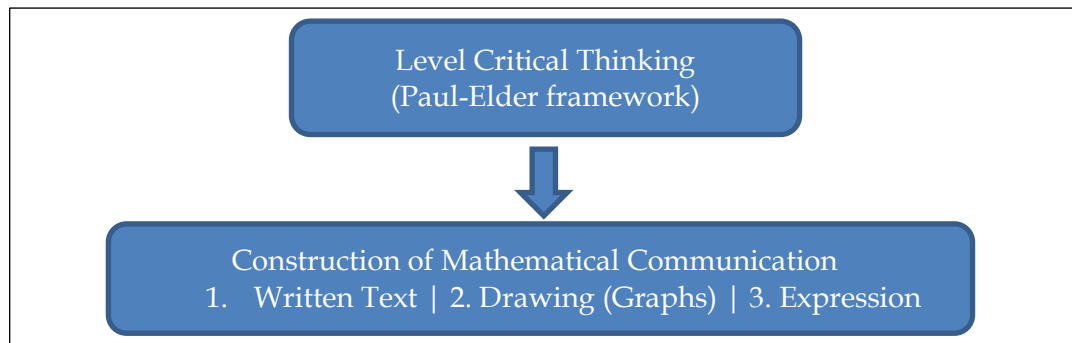


Figure 1. Conceptual Framework of Mathematical Communication Construction Based on Students' Critical Thinking Levels

Based on these theoretical gaps, this study formulated three research objectives, namely: (1) describe the distribution of the level of critical thinking of high school students in functional materials; (2) analyze the construction of students' mathematical communication skills at each level of critical thinking; and (3) identify patterns of mathematical communication constraints experienced by students at each level of critical thinking.

METHODS

This study uses a qualitative approach with a descriptive exploratory design to understand the authentic cognitive structure of students' mathematical expressions (Sugiyono, 2013). The research was carried out at Muhammadiyah Private High School 1 Medan, involving 32 students in class XI-2 who were selected through purposive sampling techniques. From this population, 9 students were selected as key informants representing three categories (3 students of high critical thinking, 3 medium students, and 3 of low critical thinking) for the purpose of data triangulation.

The research instrument has been validated by two experts with a Content Validity Ratio (CVR) value of 0.87. The mathematical communication ability test instrument consists of three question items, each of which represents a mathematical communication indicator, as follows.

Question item 1 (indicators of written communication): *"Explain using formal mathematical language and your own language the structural differences between relationships in general and functions. Give each one a real example as an explanation!"* question item 2 (image communication indicator): *"Given the linear function $f(x) = 2x + 1$ with a finite domain $\{-2, -1, 0, 1, 2\}$. Draw a complete and accurate graph of the function on the Cartesian plane, label each axis, and determine the*

range of the function!" question item 3 (mathematical expression indicator): "It is known that $f(x) = 3x - 2$ and $g(x) = x^2 + 1$. Define and formulate the composition functions $(f \circ g)(x)$ and $(g \circ f)(x)$. Write down the final result using the standard function algebraic notation!"

Data analysis was carried out systematically using the Miles, Huberman, and Saldana models through the stages of data reduction (open and axial coding), data presentation, and conclusion drawn.

RESULTS AND DISCUSSION

1. Distribution of Students' Critical Thinking Levels (Research Objective 1)

In the initial stage of this study, the basic profile of the critical thinking level of the 32 students who were the subjects of the study. The empirical distribution is shown in table 1.

Table 1. Distribution of Students' Critical Thinking Levels

No	Critical Thinking Level	Number of Students	Percentage (%)
1	High	7	21,88
2	Moderate	17	53,12
3	Low	8	25,00
	Total	32	100,00

Table 1 shows that most students are in the category of moderate critical thinking. This finding is in line with the opinion of Firdaus, Wahyudin, and Herman (2021), who stated that students at the secondary education level are generally at the beginner critical thinking stage, namely, able to apply logical analysis sporadically but have not been able to maintain systematic reflective control in completing complex mathematical tasks.

2. Construction of Mathematical Communication Skills at Every Level of Critical Thinking (Research Objective 2)

a. High Critical Thinking Level

Students at this level are able to build strong and mutually reinforcing relationships between all modes of communication. In the aspect of written communication, students are able to compile explanations with high semantic clarity. In question item 1, subject 1 (S1) writes the answer as follows: "A relationship is a pair between two sets. However, a function is a special relationship in which each element in the domain set A must be paired with exactly one element in the set of codomains B ."

In the aspect of drawing, students show excellent precision. S1 depicts the coordinate points $(-2,-3)$, $(-1,-1)$, $(0,1)$, $(1,3)$, and $(2,5)$ separately without connecting them with a straight line, which shows a deep conceptual understanding that discrete domains produce discrete plots as well. In the aspect of mathematical expressions, students show proper algebraic manipulation: $(f \circ g)(x) = 3(x^2+1) - 2 = 3x^2 + 1$.

b. Moderate Critical Thinking Level

Students at this level demonstrate semi-structured mathematical communication. On the aspect of written communication, Subject 4 (S4) wrote: *"A function is a relation in which each element on the left has a partner on the right."* This answer shows an informal and intuitive understanding, but does not use a formal academic register.

The drawing ability in this group is quite adequate, but it is prone to structural negligence; Students often draw graphs without including scales or axis labels. The biggest weakness occurs in the aspect of mathematical expression, where the notation of the composition of functions is often understood as a simple algebraic multiplication operation.

c. Low Critical Thinking Level

Students at this level exhibit highly fragmented mathematical communication. In the aspect of written communication, students' answers tend to be very ambiguous. For example, Subject 8 (S8) states: *"Functions are those that use formulas; relationships do not."*

In the drawing aspect, students fail to map the domain into the Cartesian plane precisely, and often draw intersecting lines randomly. In the aspect of mathematical expressions, students cannot operate the symbols of the composition of functions, either by leaving the answer blank or writing down irrelevant algebraic sequences.

Table 2. Examples of Student Work Results on Mathematical Expression Questions

High Level (S1)	Intermediate Level (S4)	Low Level (S8)
$f(g(x)) = 3(x^2+1) - 2$ $= 3x^2 + 1$	$f(g(x)) = f \times (x^2+1)$ $= 3x - 2 \times (x^2+1)$	$f(g(x)) = f(x) + g(x)$ $= (3x-2)+(x^2+1)$
Precise and consistent notation	Incorrect notation, composition is understood as multiplication	Not understanding the concept of composition at all

3. Patterns of Mathematical Communication Constraints (Research Objective 3)

Through a structural examination of the results of the written work and interview transcripts, this study succeeded in identifying three main types of mathematical communication constraints, as summarized in table 3.

Table 3. Profile of Communication Constraints Based on Critical Thinking Level

Indicator	High Level	Medium Level	Low Levels
Written Communication	Comprehensive, formal, and theoretically structured	Descriptive, still using a lot of colloquial language	Fragmented, highly ambiguous, or empty
Image	Precise mapping, as per the domain boundaries	Sketch a basic sketch without axial conventions	Distortion graphics system; Random Transformations
Mathematical Expressions	Consistent notation and smooth transitions	Syntax errors; Composition confused with multiplication	Experiencing structural stalemates: An Absolute Symbolic Error

a. Language Register Constraints

Students with moderate and low levels of critical thinking experience significant translational constraints when moving from informal language registers to formal mathematics registers. In the interview, S4 stated: "I know what it means in my head, but when it has to be written in the right mathematical terms, it feels like writing in a foreign language." This suggests that lower levels of critical thinking limit students' access to structured cognitive frameworks, so students tend to rely on less precise everyday vocabulary (Nurhayati & Zanthi, 2020).

b. Representational Constraints

A major obstacle occurs when students have to convert algebraic compositions into symbolic representations. An example of this error is found in the moderate critical thinking group, as seen in the following interview excerpt. The interviewer asked how S5 explained the writing step $(f \circ g)(x) = (3x$

$-2)(x^2 + 1)$. Subject 5 (S5) replied that when he saw the composition symbol $\circ$ or the form $f(g(x))$, he thought it represented multiplication, so he applied the distribution method (FOIL) to solve it.

These findings prove that these errors are not just procedural errors, but conceptual constraints where students misinterpret composition symbols as algebraic multiplication operations. This finding is in line with the results of research by Putri and Wijaya (2021), who also found similar mistakes in high school students in understanding function notation.

c. Connection Constraints

Students with low levels of critical thinking experience serious connection constraints. They treat written text, images, and algebraic notation as separate, unrelated tasks. For example, Subject 8 (S8) states that he is not aware that the graph he drew in question Item 2 should show the same coordinates as the values in the range table he wrote. This shows the absence of metacognitive verification at all in students with low levels of critical thinking (Hidayat & Husnussalam, 2023).

4. Analytical Synthesis and Discussion

The empirical evidence in this study shows that students' mathematical communication profiles are directly shaped by their critical thinking habits. Students with high levels of critical thinking show strong metacognitive control (Paul & Elder, 2020). They not only generate answers, but also consistently monitor the process of working on them. As stated by S1 in the interview, he always draws graphs first to verify his algebraic results; If the curve decreases while the equation is positive, he realizes that there is a mistake in his work. This form of self regulation is in line with Sfard (2008) theory, which emphasizes that high-level communication requires disciplined intrapersonal dialogue.

In contrast, students at moderate and low critical thinking levels do not have such an internal monitoring system. They show no signs of self correction, leading to unverified notation errors as well as inconsistent graphical representations. This finding expands on the research results of Rahmatiya and Miatun (2020) by showing that miscommunication is not just a coincidental mistake but is a structural problem caused by a lack of critical reflection, so that students are unable to recognize errors in their own mathematical expressions.

This finding is also in line with the results of research by Rahmatika, Batubara, and Sari (2022), which shows that students' mastery of mathematical representations in trigonometry materials is greatly influenced by their ability

to visualize and verify relationships between concepts independently. Thus, strengthening critical thinking skills through the habit of reflection and self-verification is an important key in building students' mathematical communication that is complete and integrated in the three modes of communication, namely writing, drawing, and symbolic expression.

CONCLUSION

This study found that students with different levels of critical thinking demonstrate different profiles of mathematical communication skills. Students with a high level of critical thinking are able to communicate systematically and accurately on all indicators, namely written text, images, and mathematical expressions, as well as use graphical representations to actively monitor and verify the results of their algebraic work. Students with a moderate level of critical thinking show adequate textual communication, but still have difficulty with symbolic notation and often swap the concept of function composition for multiplication operations. Students with low critical thinking levels show significant deficiencies in all modes of communication, characterized by unstructured written descriptions, inaccurate visual graphics, and an inability to use mathematical symbols appropriately.

The pedagogical implications of these findings suggest that teachers need to move away from isolated procedural exercises. Instead, the learning design must integrate critical thinking skills and mathematical communication simultaneously. Strategies that can be applied include the use of a collaborative problem-based learning model and the application of a structured reflection journal. These approaches are expected to help students build the metacognitive awareness needed to transition effectively from informal language to formal mathematical registers.

DECLARATIONS

Author Contribution : NA: Conceptualization, Investigation, Methodology, Instrument Development, Data Analysis, Writing-Original Draft, Supervision, and Writing-Review and Editing.
ID: Conceptualization, Methodology, Validation, Supervision, Instrument Development.

KMAF: Conceptualization, Methodology,
Validation, Supervision, Instrument Development.

Funding Statement : This research was not funded.
Conflict of Interest : The authors declare no conflict of interest.
Additional Information : Additional information is available from the
corresponding author upon reasonable request.

REFERENCES

- Afgani, M. W., & Sutawijaya, A. (2022). Students' mathematical communication skills in solving mathematical problems. *Journal on Mathematics Education*, 16(2), 145–160. <https://doi.org/10.22342/jpm.16.2.15402.145-160>.
- Firdaus, F. M., Wahyudin, W., & Herman, T. (2021). Improving primary students' mathematical literacy through problem-based learning and direct instruction. *Educational Research and Reviews*, 12(4), 212–219. <https://doi.org/10.5897/ERR2017.3220>.
- Herawaty, D., Warni, A., & Angraini, L. M. (2021). Construction of understanding of function concepts in prospective mathematics teacher students. *Mosharafa: Jurnal Pendidikan Matematika*, 10(3), 421–432. <https://doi.org/10.31980/mosharafa.v10i3.854>.
- Hidayat, W., & Husnussalam, H. (2023). Analysis of the mathematical communication skills of high school students in solving context-based statistical problems. *Journal Elemen*, 9(1), 25–41. <https://doi.org/10.29408/jel.v9i1.6981>.
- Ministry of Education and Culture. (2023). *Reflection on the results of PISA 2022 to improve the quality of Indonesian education*. Jakarta: Center for Assessment and Learning.
- Nasution, A. E., Irvan, & Coal, I. H. (2020). The application of GeoGebra-assisted problem-based learning and ethnomathematics models to improve mathematical communication skills. *Journal of Mathematics Education Sigma (JMES)*, 1(2), 55–64. <https://doi.org/10.30596/jmes.v1i2.5231>.
- Nurhayati, E., & Zanthi, L. S. (2020). Analysis of the mathematical communication skills of junior high school students in solving story problems. *Journal on Education*, 2(2), 138–147. <https://doi.org/10.31004/joe.v2i2.163>.

- Paul, R., & Elder, L. (2020). *Critical thinking: tools for taking charge of your learning and your life* (4th ed.). Upper Saddle River: Pearson Education.
- Putri, R. I. I., & Wijaya, A. (2021). The analysis of high school students' misconceptions on the topic of function was reviewed from mathematical reasoning skills. *Jurnal Riset Pendidikan Matematika*, 8(1), 87–100. <https://doi.org/10.21831/jrpm.v8i1.39198>.
- Rahmatika, A., Batubara, I. H., & Sari, I. P. (2022). The application of GeoGebra software in students' mathematical communication skills in trigonometry courses. *EduTech: Journal of Education and Social Sciences*, 8(2), 1–10.
- Rahmatiya, R., & Miatun, A. (2020). The analysis of mathematical problem-solving skills was reviewed from the critical thinking skills of junior high school students. *Theorem: Mathematical Theory and Research*, 5(2), 221–234. <https://doi.org/10.25157/teorema.v5i2.3619>.
- Sfard, A. (2008). *Thinking as communicating: human development, the growth of discourses, and mathematizing*. Cambridge University Press. <https://doi.org/10.1017/CBO9780511499944>.
- Sugiyono. (2013). *Metodologi penelitian kuantitatif, kualitatif, dan R & D*. Bandung: Bandung: Alfabeta.
- Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (2022). Teachers' teaching practices and beliefs regarding context-based tasks and their relation to students' difficulties in solving these tasks. *Mathematics Education Research Journal*, 27(1), 1–27. <https://doi.org/10.1007/s13394-014-0107-6>.
- Yildiz, P., & Baltaci, S. (2021). The examination of mathematical communication skills of secondary school students in mathematics class. *International Journal of Educational Studies in Mathematics*, 8(2), 93–108. <https://doi.org/10.11113/jt.v63.2014>.