

An Analysis of Tenth Grade Students' Numeracy Literacy After Experiencing Learning Through Problem Based Learning Flipped Classroom

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Abstract:

Numeracy and literacy in trigonometry are important for solving contextual mathematical problems and applying mathematical concepts in daily life. However, many students still have difficulties in representing angles of elevation and depression, creating visual representations, choosing appropriate trigonometric ratios, applying procedures, and writing accurate calculation results. This study aimed to describe the implementation of the Problem Based Learning Flipped Classroom model in teaching trigonometric ratios and to analyze students' Numeracy Literacy after the learning process. This research used a qualitative descriptive method involving students of class X C at Stella Duce 2 Senior High School, Yogyakarta, in the 2025/2026 academic year. Data were collected through field notes, written tests, and interviews. The results showed that students achieved the knowing level of Numeracy Literacy well, but the applying and reasoning levels still needed improvement, especially in organizing and implementing problem solving plans. Interviews with five selected students supported these findings. In conclusion, the Problem Based Learning Flipped Classroom model provided opportunities for students to develop numeracy literacy through independent learning, collaborative problem solving, and reflection activities, although additional scaffolding is still needed to strengthen applying and reasoning skills in trigonometric contexts.

Abstrak:

Kemampuan numerasi dalam trigonometri penting untuk menyelesaikan masalah matematika kontekstual dan menerapkan konsep matematika dalam kehidupan sehari-hari. Namun, banyak siswa masih mengalami kesulitan dalam merepresentasikan sudut elevasi dan depresi, membuat representasi visual, memilih perbandingan trigonometri yang tepat, menerapkan prosedur penyelesaian, serta menuliskan hasil perhitungan secara akurat. Penelitian ini bertujuan untuk mendeskripsikan implementasi model pembelajaran Problem Based Learning Flipped Classroom pada materi perbandingan trigonometri dan menganalisis kemampuan numerasi siswa setelah mengikuti proses pembelajaran. Penelitian ini menggunakan metode deskriptif kualitatif dengan subjek siswa kelas X C SMA Stella Duce 2 Yogyakarta tahun ajaran 2025/2026. Data dikumpulkan melalui catatan lapangan, tes tertulis, dan wawancara. Hasil penelitian menunjukkan bahwa kemampuan numerasi

siswa pada level knowing telah tercapai dengan baik, sedangkan level applying dan reasoning masih perlu ditingkatkan, terutama dalam menyusun dan mengimplementasikan rencana penyelesaian masalah. Hasil wawancara dengan lima siswa yang dipilih untuk mengklarifikasi temuan tersebut. Dengan demikian, model pembelajaran Problem-Based Learning Flipped Classroom dapat mendukung pemahaman siswa terhadap materi perbandingan trigonometri, meskipun masih diperlukan pendampingan lebih lanjut untuk memperkuat kemampuan berpikir tingkat tinggi siswa.

Keywords:

Numeracy Literacy, Problem Based Learning, Flipped Classroom,
Trigonometric Ratios

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INTRODUCTION

Numeracy skills are essential competencies in the 21st century because they support logical thinking, problem solving, and decision making in daily life. The Organisation for Economic Co-operation and Development, OECD (2019) emphasizes that numeracy is a key prerequisite for mathematics learning in the Industrial Revolution 4.0 era, equipping individuals with functional literacy. Numeracy requires students to be able to perform mathematical modeling to meaningfully apply concepts in a variety of complex contextual situations. In line with this, Verschaffel, Schukajlow, Star, and Van Dooren (2020) emphasize that developing modeling skills from an early age is crucial so that students are not only skilled in computational procedures but also able to authentically connect mathematics to reality. This is supported by Rakhmawati and Mustadi (2022), who explain that numeracy literacy is crucial in supporting conceptual mastery through relevant mathematical applications.

One mathematics topic that consistently presents a significant barrier to students' numeracy development is trigonometric ratios. At the tenth-grade level, students are in a critical transition phase from arithmetic thinking to geometric reasoning, which requires advanced spatial visualization skills to understand the relationship between angles and side ratios in right triangles. Gur (2009) explains that students' primary difficulties often stem from a failure

to connect abstract trigonometric concepts to physical representations, leading them to engage in flawed decision making when determining relevant ratios. This issue is exacerbated by students' reliance on rote memorization of sine, cosine, and tangent formulas, which bypasses functional conceptual understanding. Similarly, Challenger (2009) emphasizes the gap between procedural and conceptual knowledge, where students may memorize formulas but fail to use them as problem solving tools in real world contexts. In the context of numeracy, this barrier manifests itself when students are faced with problems involving angles of elevation and depression. Many students struggle to construct accurate visual models from textual descriptions, which Hansen (2020) argues is an indicator of poor mathematical visual literacy. As a result, students often misplace variables in mathematical models or use inappropriate operations. Furthermore, Webber (2016) states that without an understanding of the nature of constant ratios, students struggle to validate the reasonableness of their solutions. This limitation leads students not only to technical errors in calculations but also to failure at the cognitive level of reasoning, the stage where they should be able to accurately evaluate and communicate mathematical solutions in everyday contexts.

To address these cognitive barriers and low numeracy literacy, a transformation of learning models is needed that shifts the focus from procedural memorization to meaningful problem solving. One relevant approach is the Problem Based Learning Flipped Classroom (PBL-FC) model, which strategically integrates independent learning and collaborative problem solving. In this model, students master basic concepts (level of knowledge) independently before class sessions through digital materials, thus fully optimizing classroom interactions for in depth discussions and contextual problem-solving. Bergmann and Sams assert that flipped classrooms maximize educational interaction by moving direct instruction outside the classroom, which, in the context of trigonometry, provides students with more time to practice their spatial visualization (Gasparic, 2017). Furthermore, the integration of PBL within the flipped classroom framework creates a learning environment that supports the development of higher order Numeracy Literacy. Chis, Moldovan, Murphy, Pathak, and Muntean (2018) state that this synergy facilitates active learning through collaboration that requires students to apply concepts to real-world situations. According to Lo and Hew (2017), this model is very effective in improving problem-solving skills because students come to class with better cognitive readiness, allowing teachers to focus task design on

the application and reasoning levels. Akcayir and Akcayir (2018) added that flipped learning can reduce students' cognitive load when facing complex material such as trigonometry, because they have the control to repeat the basic material independently. Thus, the transition from theoretical understanding to the application of numeracy emphasized by Verschaffel, Schukajlow, Star, and Van Dooren (2020) can be facilitated through PBL activities that challenge students to construct and validate their own mathematical models.

Several previous studies have documented the effectiveness of the flipped classroom model in improving students' numeracy skills and mathematical understanding. Lestari, Nugraheni, and Kurniasih (2023) found that students taught using the flipped classroom model achieved better numeracy performance compared to those taught using conventional learning. Similar findings were reported by Salsabila, Lubis, and Narpila (2024); Khamdani, Rasiman, and Sulianto (2023), who demonstrated that the flipped learning environment significantly supports the development of numeracy literacy. However, most of these studies tend to evaluate learning outcomes through quantitative measures at a macro level, often overlooking the intricate nuances of students' internal thought processes. There remains a critical research gap in providing an in-depth qualitative examination of students' numeracy ability profiles based on hierarchical cognitive domains—namely, knowing, applying, and reasoning as defined by the TIMSS framework (Mullis & Martin, 2017). This gap is particularly evident in the topic of trigonometric ratios, where students often struggle with the transition from instrumental to relational understanding (Skemp, 1976; Vinner, 1991). Therefore, this study aims to describe the implementation of the PBL-FC model in teaching trigonometric ratios and to provide a comprehensive qualitative analysis of students' numeracy profiles after participating in the process. The novelty of this research lies in its focus on deconstructing students' reasoning processes at each cognitive level, offering a more granular perspective than previous performance based studies. The findings are expected to provide practical contributions for mathematics teachers in designing targeted numeracy based interventions and theoretical contributions to curriculum development aligned with the Minimum Competency Assessment (AKM) and 21st-century global challenges.

METHODS

This study applies a qualitative approach with a descriptive design to describe the implementation of the Problem based Learning Flipped Classroom

(PBL-FC) model and analyze the profile of students' numeracy abilities on the topic of Trigonometric Ratios. The qualitative descriptive approach was chosen because this study aims to provide a systematic, factual, and in depth description of students' thinking processes in solving contextual trigonometric problems. The study was conducted in a private high school in Yogyakarta (hereinafter referred to as SMA X) in the 2025/2026 academic year, involving 35 grade X students as participants. The selection of subjects was carried out using a purposive sampling technique, where the class was chosen because students had mastered the prerequisite material related to right triangles and had diverse mathematical abilities that supported the analysis of variations in numeracy profiles at the knowing, applying, and reasoning levels.

The learning process was conducted using the Problem based Learning Flipped Classroom (PBL-FC) model over four meetings (2×45 minutes each) through three phases: pre class, in class, and post class activities. Students independently studied learning materials before class, participated in collaborative problem solving activities during face to face sessions, and completed reflection and reinforcement activities after class. Contextual trigonometry problems were used throughout the learning process to support the development of students' numeracy literacy. In the third meeting, PhET Interactive Simulations were integrated to support students' understanding of the unit circle by allowing them to visualize the relationship between angles, coordinates, and trigonometric values across different quadrants.

Research data were collected through field notes, written tests, and clinical interviews. Field notes were used to record the dynamics of learning activities and obstacles that emerged during the implementation of the PBL-FC model. The written test instrument consisted of three content valid contextual essay questions to measure students' Numeracy Literacy on the topic of Trigonometric Ratios. Subsequently, structured interviews were conducted to confirm students' written responses and explore their reasoning processes in more depth. Interview subjects were selected using a purposive sampling technique based on the written test results using grouping criteria adapted from Arikunto (2021), namely based on the average (mean) score and standard deviation. From the grouping results, five subjects were selected representing the diversity of numeracy profiles, consisting of two students in the high category, one student in the medium category, and two students in the low category. This selection ensures a comprehensive representation of various

cognitive obstacles and reasoning patterns across the three numeracy levels (Creswell & Poth, 2018).

Table 1. Criteria for Selecting Interview Subjects

Categories	Indicator	Score Range
Low	Score lower than the mean minus the standard deviation	$x < (\bar{x} - s)$
Medium	Score between the mean $\pm$ standard deviation	$(\bar{x} - s) \leq x \leq (\bar{x} + s)$
High	Score higher than the mean plus the standard deviation	$x > (\bar{x} + s)$

The main research instruments in this study included field notes, a numeracy test, and a structured interview guide. To ensure the quality of the instruments, content validity using expert judgment was conducted by two mathematics education experts to ensure alignment between the test items and the cognitive indicators of numeracy. Data credibility was ensured through methodological triangulation by comparing the results of written test and interviews from the same subjects. This approach was used to verify the consistency of students' responses and obtain a more comprehensive understanding of their numeracy literacy (Creswell & Poth, 2018; Moleong, 2017).

Data were analyzed using the interactive models of Miles, Huberman, & Saldana (2014), which consist of three stages: data condensation, data display, and conclusion drawing and verification. Data from written tests, interviews, and field notes were organized and interpreted to identify students' numeracy literacy profiles at the knowing, applying, and reasoning levels based on the Minimum Competency Assessment framework (Kemdikbud, 2017). The numeracy indicators used in this study are presented in table 2.

Table 2. Numeracy Skill Indicators

No	Numeracy Indicator	Numeracy Level
1	Representing problem information in a formal mathematical sketch/diagram.	Knowing
2	Identifying and extracting relevant data (knowns and unknowns) from contextual descriptions.	Knowing
3	Selecting and executing appropriate trigonometric concepts and formulas for specific contexts	Applying

No	Numeracy Indicator	Numeracy Level
4	Determining the most efficient trigonometric ratios to solve right triangle problems	Applying
5	Performing accurate calculations to determine lengths, heights, or angles in real-world scenarios	Applying
6	Justifying the choice of a specific trigonometric strategy based on the given contextual constraints	Reasoning
7	Analyzing and linking multiple mathematical elements to solve complex or non-routine contextual problems	Reasoning
8	Evaluating the logical consistency and reasonableness of the final solution within the original context	Reasoning

Although eight numeracy indicators were used in the instrument design, these indicators were grouped into five broader analytical categories to facilitate the presentation and interpretation of the findings. These categories reflect the stages of students' mathematical problem-solving processes, including visual representation, identifying relevant information, planning strategies, implementing solutions, and evaluating final answers. The correspondence between the eight numeracy indicators and the five analytical categories is presented in table 3.

Table 3. Mapping of Eight Numeracy Indicators into Five Analytical Categories

No	Numeracy Indicator	Numeracy Level	Analytical Category
1	Representing problem information in a formal mathematical sketch/diagram.	Knowing	Representing problem situations visually
2	Identifying and extracting relevant data (knowns and unknowns) from contextual descriptions.	Knowing	Identifying known and asked information
3	Selecting and executing appropriate trigonometric concepts and formulas for specific contexts	Applying	Planning problem-solving strategies

No	Numeracy Indicator	Numeracy Level	Analytical Category
4	Determining the most efficient trigonometric ratios to solve right triangle problems	Applying	
5	Justifying the choice of a specific trigonometric strategy based on the given contextual constraints	Reasoning	
6	Performing accurate calculations to determine lengths, heights, or angles in real-world scenarios	Applying	Implementing the solution plan
7	Analyzing and linking multiple mathematical elements to solve complex or non-routine contextual problems	Reasoning	Implementing the solution plan
8	Evaluating the logical consistency and reasonableness of the final solution within the original context	Reasoning	Checking and evaluating the final answer

As shown in table 3, the eight numeracy indicators were consolidated into five broader analytical categories to facilitate data presentation while maintaining alignment with the AKM numeracy framework and the original research instrument.

RESULTS AND DISCUSSION

Before the first meeting, a diagnostic assessment was conducted to identify students' prior knowledge related to right triangles, the Pythagorean theorem, and basic numeracy literacy. The assessment consisted of five multiple choice questions completed by 35 students.

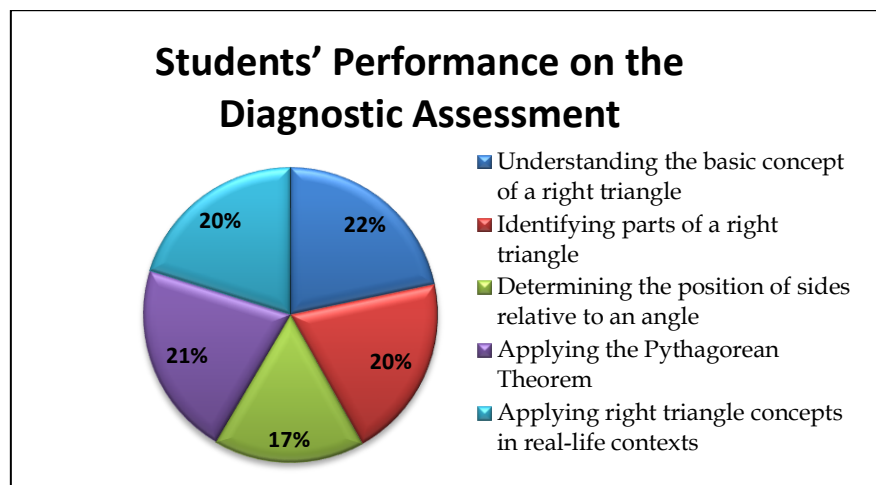


Figure 1. Problem Understanding Stage Completion Percentage

Figure 1 illustrates the distribution of students' performance across five prerequisite competencies related to right triangles. The largest proportion of correct responses was observed in understanding the basic concept of a right triangle (22%), followed by applying the Pythagorean Theorem (21%). Meanwhile, identifying parts of a right triangle and applying right-triangle concepts in real-life contexts each contributed 20% of the total correct responses. The lowest achievement was found in determining the position of sides relative to an angle (17%), indicating that students had greater difficulty with spatial and relational aspects of right triangle concepts. Overall, the results demonstrate that students had acquired basic prerequisite knowledge; however, additional reinforcement was still required, particularly in understanding angle-side relationships, visualizing right triangles, and applying trigonometric concepts meaningfully. Therefore, the first learning meeting was designed to strengthen conceptual understanding through independent learning before class and collaborative problem solving during classroom activities.

1. Implementation of the Learning Process

The implementation of the Problem-Based Learning Flipped Classroom (PBL-FC) model in Class X C at SMA Stella Duce 2 Yogyakarta was carried out in three main phases: pre class, in class, and post class activities. Based on field notes, in the pre class phase, students were provided with instructional videos and teaching materials through Google Classroom to help them build an initial understanding of trigonometric ratios independently before attending face to face learning sessions. During the in class phase, learning activities followed the syntax of problem based learning, beginning with problem orientation through

contextual problems related to ladders, towers, angles of elevation, and height differences. Students were then organized into small groups to collaboratively discuss and investigate the problems. Classroom observations showed that most students actively participated in group discussions by identifying known information, asking questions, drawing sketches, and determining appropriate trigonometric ratios. Interactions during discussions also reflected processes of conceptual clarification and improvement of students' understanding of the given problems. The following dialogue illustrates this process:

- S1 : "For this second problem, the angle of elevation is above, right, Ma'am?"
- Teacher : "Try to look again at the information in the problem. The angle is formed from the observer's eye when looking at the top of the siren tower. So, where is the angle actually formed?"
- S1 : "From the eye to the horizontal line, right, Ma'am?"
- Teacher : "Correct, please continue working on it."

The dialogue shows that the teacher acted as a facilitator by providing guiding questions so that students could discover concepts independently. In addition to group discussions and teacher guidance, PhET Interactive Simulations were used in the third meeting to help students visualize the relationship between angles, coordinates, and trigonometric values in the unit circle. Through the simulation, students could directly observe changes in sine and cosine values across different quadrants, which helped reduce misconceptions related to angle direction, quadrant positions, and the signs of trigonometric ratios. However, field notes indicated that some students initially experienced difficulties in connecting the visual representations shown in the simulation with formal trigonometric concepts and mathematical notation. Therefore, teacher guidance remained important in helping students interpret the simulation results and relate them to problem-solving activities. In the presentation stage, students presented their solutions in front of the class and responded to feedback from other groups. Finally, in the post-class phase, students carried out reflection activities and completed follow-up tasks to reinforce conceptual understanding and evaluate the problem-solving process they had undertaken.

2. Results of Students' Numeracy Ability Test

After students participated in learning using the Problem-based Learning Flipped Classroom model, the researcher conducted a written test to

measure students' Numeracy Literacy in Trigonometry Comparison material. The test consisted of three contextual essay questions developed based on eight numeracy indicators. Based on the written test results, students' numeracy profiles were categorized into high, medium, and low groups using the criteria proposed by Arikunto (2018) based on the mean score and standard deviation. Students in the high category generally demonstrated good abilities in representing problems visually, selecting appropriate trigonometric concepts, implementing solution procedures correctly, and evaluating their answers. Students in the medium category were able to identify problem information and determine solution strategies, but still experienced difficulties in calculation accuracy and reasoning consistency. Meanwhile, students in the low category tended to experience difficulties in selecting suitable trigonometric ratios, organizing solution steps, and evaluating final answers. This categorization helped the researcher analyze variations in students' numeracy abilities more comprehensively and select representative interview subjects from each category.

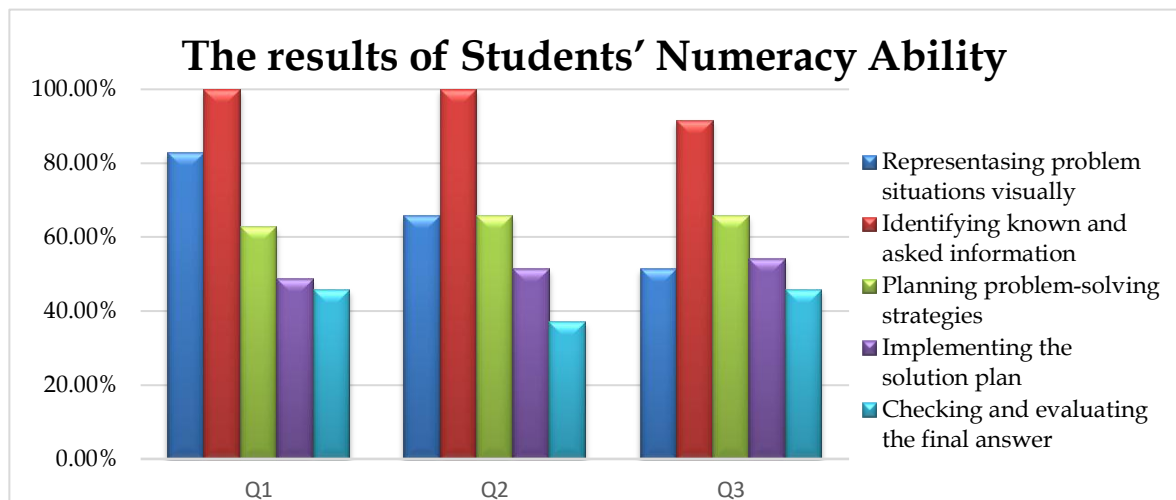


Figure 2. Students' Achievement Across Five Numeracy Analytical Categories

Based on the data presented in figure 2, there is a clear downward trend in student performance as cognitive demands increase from basic recall to higher order thinking. Most students demonstrated strong performance in the knowing level, particularly in identifying known and asked information, whereas lower achievement was observed at the applying and reasoning levels. The lowest percentages were found in implementing problem-solving plans and

evaluating final answers. These findings indicate that students generally possess basic conceptual understanding but still encounter difficulties when required to independently select strategies, implement mathematical procedures, and evaluate their solutions in contextual problems. Since this study did not compare students' performance before and after the implementation using equivalent indicators, these findings should be interpreted as a description of students' numeracy literacy after participating in PBL-FC learning rather than evidence of the model's effectiveness. To further understand students' cognitive processes and difficulties, a detailed qualitative analysis of selected research subjects is presented in the following section.

The recapitulation of students' numeracy performance across the three contextual essay questions is presented in figure 2. The data show variations in students' achievement across the five indicators of numeracy literacy. To further explore these findings, the researcher analyzed students' written work and interview results. Although five students from different achievement categories were interviewed, Subject 2 and Subject 5 were selected for detailed presentation because they represented contrasting numeracy profiles and provided clear illustrations of students' strengths and difficulties across the knowing, applying, and reasoning levels. Subject 2 represented students who demonstrated basic understanding but encountered difficulties in strategy selection and implementation, whereas Subject 5 represented students who were able to complete all stages of problem solving.

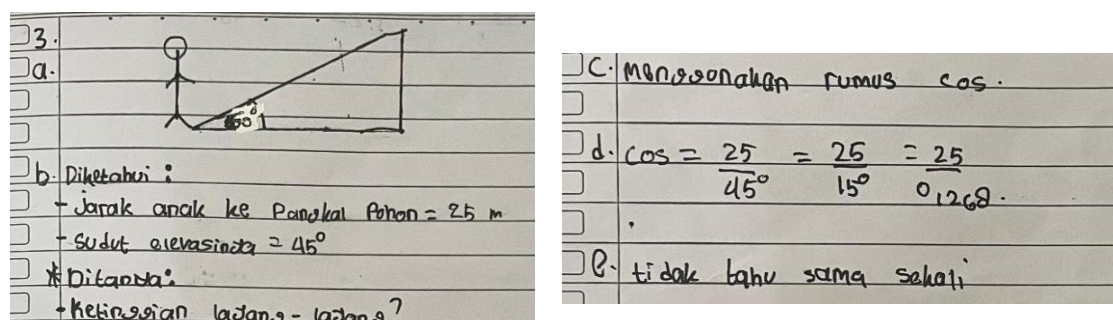


Figure 3. Subject 2's Written Solution for Question 3

A representative example of student work with partial understanding is shown by Subject 2 in solving problem number 3. The student was able to represent the situation by drawing a right triangle that illustrates the position of a child, the tree, a horizontal distance of 25 m, and an angle of elevation of 45°, although the sketch did not fully represent the kite above the tree as the main

object of the problem. This is reflected in the student's statement, "Because there is a child and a tree." The student also correctly identified the known information and the question being asked, namely the height of the kite above the tree. In the planning stage, the student attempted to use trigonometric ratios by selecting the cosine function; however, the student was unable to justify this choice, as indicated by the response, "I don't know." In the implementation stage, the student performed calculations using cosine but applied incorrect procedures, such as dividing values without considering the correct trigonometric relationships, resulting in an unclear and incorrect final answer. These findings indicate that Subject 2 successfully achieved the knowing level by representing problem situations visually and identifying relevant information. However, difficulties were still observed at the applying and reasoning levels, particularly in selecting appropriate trigonometric concepts, implementing solution procedures, justifying the chosen strategies, and evaluating the final answer.

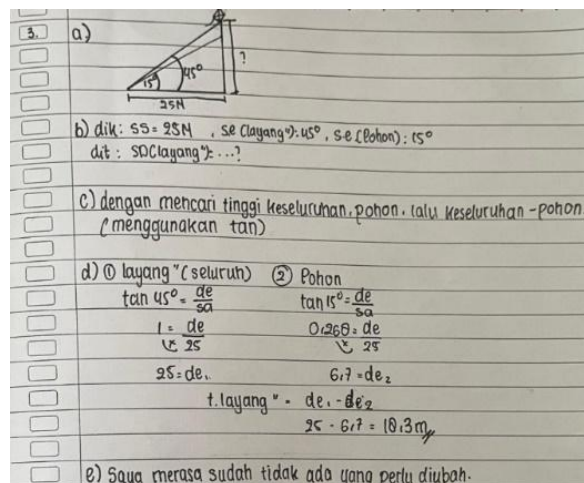


Figure 4. Subject 5's Written Solution for Question 3

A representative example of student work is shown by Subject 5 in solving problem number 3. The student was able to represent the problem situation visually by constructing two right triangles with the same base of 25 m, indicating angles of elevation of 15° toward the top of the tree and 45° toward the kite, as reflected in the statement, "Because the tree stands perpendicular to the ground and the line of sight forms the hypotenuse, it creates a right triangle."

The student also correctly identified the known and unknown information, including the horizontal distance and both angles of elevation, as well as the height of the kite above the tree as the quantity to be determined. In

the planning stage, the student proposed an appropriate strategy by determining the total height and the tree height separately using trigonometric ratios, then subtracting both values. This plan was implemented consistently using the tangent ratio, resulting in a total height of 25 m and a tree height of approximately 6.7 m, as explained by the student: *"I used $\tan 45^\circ$, which equals 1, so the total height is 25 meters, then $\tan 15^\circ$ times 25 gives about 6.7 meters."*

The final result was obtained by subtracting these values to find the height of the kite above the tree. During the evaluation stage, the student stated, *"I checked it again,"* and confirmed that no corrections were needed. These findings indicate that Subject 5 successfully fulfilled all eight numeracy indicators, including representing problem situations, identifying relevant information, selecting trigonometric concepts, determining appropriate ratios, performing calculations, justifying strategies, integrating mathematical information, and evaluating the final solution.

The findings of this study can be interpreted based on the numeracy framework of the Minimum Competency Assessment (AKM), which consists of the knowing, applying, and reasoning levels. At the knowing level, students showed the highest achievement, particularly in identifying known and asked information, with percentages reaching 100% in Questions 1 and 2 and 91.43% in Question 3. This indicates that most students were able to understand contextual problem situations and recognize important information presented in the questions. The success at this level may be influenced by the pre-class activities in the Problem Based Learning Flipped Classroom (PBL-FC) model, where students studied instructional videos and teaching materials independently before classroom learning. Through these activities, students obtained an initial conceptual understanding of trigonometric ratios, making it easier for them to identify problem information during the written test. This finding supports the view that flipped classroom learning can strengthen students' basic conceptual understanding before entering collaborative classroom activities.

At the applying level, students began to experience difficulties, particularly in selecting appropriate trigonometric ratios such as sine, cosine, and tangent to solve contextual problems. Although more than half of the students were able to plan problem solving strategies, many students still chose incorrect ratios or could not explain the reasons for selecting certain concepts. This was reflected in Subject 2, who used the cosine ratio but responded, *"I don't know,"* when asked about the reason for choosing the formula. In contrast,

Subject 5 was able to explain the relationship between known and unknown sides and correctly used the tangent ratio. These findings indicate that many students were still relying on memorization of formulas rather than conceptual understanding of trigonometric relationships. In this stage, the role of the teacher during in-class activities became very important. Through guiding questions and classroom discussions, the teacher acted as a facilitator to help students who were still “stuck” in determining appropriate strategies and connecting contextual situations with mathematical concepts.

At the reasoning level, students showed the lowest performance, especially in checking and evaluating their answers, with percentages ranging from 37.14% to 45.71%. This finding indicates that many students were not yet accustomed to reflecting on their problem solving processes or verifying whether their answers were reasonable. Most students tended to stop once they obtained a result without rechecking calculations or reconsidering the suitability of the strategy used. Subject 2, for example, was unable to identify mistakes in the solution even after checking the answer, while Subject 5 demonstrated reflective thinking by confidently re evaluating the calculations performed. The low achievement at this level may be related to students’ limited habit of reflection in mathematics learning, where classroom activities often emphasize obtaining final answers rather than evaluating the reasoning process. Therefore, reflection activities in the post class phase of the PBL-FC model are important to strengthen students’ reasoning skills and awareness of errors in solving contextual problems.

The implementation of technology through PhET Interactive Simulations in the third meeting also supported students’ conceptual understanding, particularly in learning trigonometric ratios in the unit circle. Through the simulation, students could directly observe changes in sine and cosine values across different quadrants and visualize the relationship between angles, coordinates, and trigonometric values dynamically. This helped students reduce misconceptions related to angle direction, quadrant positions, and positive or negative signs of trigonometric ratios, which are often difficult to understand through static images alone. The integration of technology in the PBL-FC model therefore supported more meaningful and interactive learning experiences, especially for abstract trigonometric concepts.

Overall, the findings of this study are consistent with previous research regarding the implementation of the Problem-Based Learning Flipped Classroom (PBL-FC) model in mathematics learning. The active classroom

discussions observed during the learning process are consistent with the ideas proposed by Bergmann and Sams, who argue that flipped classroom learning promotes classroom interaction because students obtain initial conceptual understanding before entering the classroom. In this study, students were actively involved in identifying information, discussing strategies, and collaboratively solving contextual trigonometry problems during in-class activities. In addition, the difficulties experienced by students at the reasoning level are in line with the findings of Nuringtyas and Setyaningsih (2023), which indicate that students' mathematical literacy and higher order thinking skills in contextual problems are still relatively low, particularly in interpreting situations, evaluating answers, and reflecting on problem-solving processes.

The results revealed that students demonstrated stronger performance at the knowing level, while applying skills were beginning to develop, although many students still experienced difficulties in evaluating solutions and explaining their reasoning systematically. This indicates that for complex mathematical topics such as trigonometric ratios, independent learning through videos and collaborative discussions alone may not be sufficient to fully support the development of higher order numeracy skills. Therefore, teachers need to provide more structured guidance through reflective questioning, step by step problem solving support, and activities that encourage students to critically re-evaluate their answers. The novelty of this study lies in the analysis of students' numeracy skills based on the knowing, applying, and reasoning levels within the context of trigonometric ratio learning implemented through the PBL-FC model.

CONCLUSION

Based on the results of the analysis and discussion, this study concludes that the implementation of the Problem based Learning Flipped Classroom model in trigonometric ratio learning, through pre class, in class, and post class activities, supported active and student centered learning. The findings show that students' numeracy skills were strongest at the knowing level, while the applying and reasoning levels were still developing, particularly in selecting appropriate strategies, implementing procedures correctly, and evaluating solutions. These findings indicate that although the PBL-FC model supported students' conceptual understanding and engagement, additional scaffolding is still needed, especially in reasoning activities. Therefore, teachers are encouraged to provide more intensive guidance during the problem-solving

process, such as using leading questions in worksheets, guided discussions, and reflection activities to help students connect concepts, select appropriate trigonometric ratios, and re-evaluate their answers critically. In addition, the use of interactive media such as PhET Interactive Simulations can support students in visualizing abstract trigonometric concepts more effectively. This study was limited to one class and focused only on trigonometric ratio material using a qualitative descriptive approach, so the findings cannot be generalized broadly. Therefore, future research is recommended to implement the PBL-FC model in different mathematical topics, educational levels, or research designs to further explore students' numeracy skills and the effectiveness of scaffolding strategies in developing, applying, and reasoning abilities.

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DECLARATIONS

- Author Contribution : AFDA: Conceptualization, Methodology, Instrument Development, Data Collection, Data Analysis, Learning Implementation, Writing-Original Draft, and Writing-Review & Editing.
HJ: Conceptualization, Methodology, Instrument Validation, Supervision, and Writing-Review & Editing.
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- Conflict of Interest : The authors declare that there are no competing interests related to the publication of this article.
- Additional Information : The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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