MaPan : Jurnal Matematika dan Pembelajaran
p-ISSN: 2354-6883 ; e-ISSN: 2581-172X
Volume 14, No 1, June 2026 (85-105)
DOI: https://doi.org/10.24252/mapan.2026v14n1a5

Exploring Students' Critical Thinking Skills and Mathematical Communication Skills in Solving Statistical Problems

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Received June 1, 2026; Revised June 19, 2026; Accepted June 21, 2026
Available Online June 28, 2026

Abstract:
Mathematical critical thinking and mathematical communication are important competencies in solving statistical problems. However, studies examining both abilities simultaneously in the context of statistical problem solving remain limited. This study aimed to explore students' mathematical critical thinking and mathematical communication skills in solving statistical problems. A descriptive design combining quantitative and qualitative analyses was employed. The participants were 24 eighth grade students from a junior high school in Cirebon. Data were collected through statistical problem solving tests developed based on indicators of mathematical critical thinking and mathematical communication. Quantitative analysis was used to describe students' achievement on each indicator, while qualitative analysis was conducted through a cross case analysis of three students representing high, medium, and low categories. The results showed that students' mathematical critical thinking and mathematical communication skills varied across ability categories. High category students demonstrated stronger abilities in analyzing information, providing logical reasons, evaluating solutions, and communicating mathematical ideas through symbolic, visual, and verbal representations. In contrast, students in the lower category tended to focus on procedural calculations and experienced difficulties in explaining reasoning and interpreting results. These findings indicate that students with stronger critical thinking skills generally exhibited more complete and meaningful mathematical communication. Further analysis revealed similar patterns in students' mathematical critical thinking and mathematical communication across the three ability categories. Students with stronger critical thinking performance generally demonstrated clearer, more systematic, and more meaningful mathematical communication. These findings highlight the importance of developing both abilities in an integrated manner within mathematics learning.

Abstrak:
Pemikiran kritis matematika dan komunikasi matematika merupakan kompetensi penting dalam memecahkan masalah statistik. Namun, studi yang meneliti kedua kemampuan secara bersamaan dalam konteks pemecahan masalah statistik tetap terbatas. Penelitian ini bertujuan untuk mengeksplorasi pemikiran kritis matematis dan keterampilan komunikasi matematis siswa dalam memecahkan masalah statistik. Desain deskriptif yang menggabungkan analisis deskriptif dan kualitatif kuantitatif

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digunakan. Peserta tersebut adalah 24 siswa kelas delapan dari SMP di Cirebon. Data dikumpulkan melalui tes pemecahan masalah statistik yang dikembangkan berdasarkan indikator pemikiran kritis matematis dan komunikasi matematis. Analisis kuantitatif digunakan untuk menggambarkan prestasi siswa pada masing-masing indikator, sedangkan analisis kualitatif dilakukan melalui analisis lintas kasus terhadap tiga siswa yang mewakili kategori tinggi, sedang, dan rendah. Hasil penelitian menunjukkan bahwa pemikiran kritis matematis dan keterampilan komunikasi matematika siswa bervariasi di seluruh kategori kemampuan. Siswa kategori tinggi menunjukkan kemampuan yang lebih kuat dalam menganalisis informasi, memberikan alasan logis, mengevaluasi solusi, dan mengkomunikasikan ide-ide matematika melalui representasi simbolik, visual, dan verbal. Sebaliknya, siswa di kategori bawah cenderung fokus pada perhitungan prosedural dan mengalami kesulitan dalam menjelaskan penalaran dan menafsirkan hasil. Temuan ini menunjukkan bahwa siswa dengan keterampilan berpikir kritis yang lebih kuat umumnya menunjukkan komunikasi matematis yang lebih lengkap dan bermakna. Analisis lebih lanjut mengungkapkan pola serupa dalam pemikiran kritis matematis siswa dan komunikasi matematika di tiga kategori kemampuan. Siswa dengan kinerja berpikir kritis yang lebih kuat umumnya menunjukkan komunikasi matematika yang lebih jelas, lebih sistematis, dan lebih bermakna. Temuan ini menyoroti pentingnya mengembangkan kedua kemampuan tersebut secara terintegrasi dalam pembelajaran matematika.

Keywords:
Mathematical Critical Thinking Skills, Mathematical Communication Skills, Statistical Problem Solving

How to cite: Izzati, N., & Suryadi, D. A. (2026). Exploring Students' Critical Thinking Skills and Mathematical Communication Skills in Solving Statistical Problems. *MaPan: Jurnal Matematika dan Pembelajaran*, *14*(1), 85-105. https://doi.org/10.24252/mapan.2026v14n1a5.

INTRODUCTION

In the realm of learning mathematics, students still face various difficulties when solving statistical problems that require high level thinking skills. In contrast to solving procedural mathematical problems, which generally focus on applying algorithms to get the final correct answer, solving statistical problems requires students to understand data driven information, perform quantitative interpretations, construct logical arguments, evaluate the adequacy of the reasons used, and formulate mathematically justifiable conclusions. This complexity suggests that students' success in solving statistical problems depends not only on mastery of concepts and computational skills but is also influenced by critical thinking and mathematical communication skills, which

work simultaneously during the problem solving process. This situation becomes important in learning mathematics because weaknesses in one of these skills can hinder the quality of students' reasoning in understanding and solving problems that require in-depth analysis. Previous studies have shown that critical thinking skills contribute greatly to the quality of students' math problem solving. Wang and Abdullah (2024) report that learning strategies that systematically train activities such as analysis, evaluation, and reflection can significantly improve students' academic achievement.

In the context of mathematics education, critical thinking skills are understood as reflective and rational thinking processes that allow students to analyze problems, evaluate information, build arguments, and make decisions based on logical reasoning (Dolapcioglu & Doganay, 2020). In addition, mathematical communication skills also play an important role in helping students organize ideas, present mathematical information, and explain the reasoning process systematically. Research shows that students with strong mathematical communication skills tend to show better understanding of concepts because they can present problem-solving strategies in a structured and logical way (Khoirunnisya, Yurike, Susanti, Sari, & Putri, 2024). These findings show that critical thinking skills and mathematical communication are two basic competencies that support students' success in math problem-solving activities.

Although previous research has extensively examined students' mathematical critical thinking skills and mathematical communication skills, most studies still treat these two competencies as separate variables, especially in studies that focus on the effectiveness of specific learning models (Andini & Rakhmawati, 2025; Khoirunnisya et al., 2024). Research that specifically looks at how these two abilities are interrelated when students solve statistical problems is still relatively limited. In addition, most previous studies have used aggregate quantitative analysis, which only provides an overview of students' skill levels, so it has not been able to explain how weaknesses in certain critical thinking processes are related to weaknesses in mathematical communication that arise at the same problem solving stage. As a result, until now, the understanding of the pattern of relationships between these indicators is still limited.

This gap is important because learning that focuses on developing one skill separately may not enable students to cope with the complexity of the thinking process that actually occurs when students are faced with math problems that require high level thinking. This research is based on a conceptual

framework that sees mathematical critical thinking skills and mathematical communication as two interrelated competencies in the problem-solving process. In this study, mathematical critical thinking refers to the framework developed by Robert H. Ennis, which operationalizes critical thinking through five indicators: analyzing problems, making arguments, proving, drawing conclusions, and evaluating solutions. Meanwhile, mathematical communication skills refer to standards developed by the National Council of Teachers of Mathematics, which sees mathematical communication as the ability to organize, represent, convey, and systematically interpret mathematical ideas. These two competencies are considered conceptually interrelated because the mathematical reasoning process requires communication skills so that the problem solving strategies developed can be expressed clearly and logically. To see the relationship between these two competencies more deeply, this study uses an indicator based analysis framework developed by the researcher by mapping the conceptual connection between the indicators of mathematical critical thinking skills and the indicators of mathematical communication based on the similarity of cognitive demands at each stage of problem solving.

The novelty of this research lies in the use of an indicator based analysis framework to identify the relationship between the two abilities in more detail at the level of students' thinking processes. The findings of this study are expected to contribute to the development of mathematics learning that is more oriented towards strengthening critical reasoning and mathematical communication skills in the context of statistical learning. In contrast to previous studies that generally examined mathematical critical thinking skills or mathematical communication separately, this study explored the two abilities simultaneously in the context of statistical problem solving and analyzed their relationship patterns through cross-case analysis in students with different skill categories.

METHODS

1. Research Design

This study uses a qualitative descriptive design supported by descriptive quantitative analysis. The qualitative descriptive approach is used because it allows researchers to describe and interpret phenomena contextually based on the experience and characteristics of data obtained directly from participants (Doyle, McCabe, Keogh, Brady, & McCann, 2020). This study aims to explore

and describe students' mathematical critical thinking skills and mathematical communication skills in solving statistical problems, as well as to identify the tendency of correlation between the two abilities based on the characteristics of students' answers.

The qualitative approach is used as the main orientation of the research because this study focuses on the interpretation of students' written answers and the identification of thinking patterns, communication, and problem-solving processes demonstrated by students. Meanwhile, quantitative data is used as supporting data to provide an overall picture of student achievement on each indicator and to help select representative subjects for more in-depth analysis.

2. Research Subject

This study involved 24 eighth grade students from Junior High School SMP Negeri 15 Cirebon. Since all students in the class participated in the study, a total sampling technique was employed. All students completed the critical thinking and mathematical communication tests to provide an overall description of their abilities. Based on the results of the descriptive analysis, three students representing high, medium, and low achievement levels were selected through stratified purposive (criterion based) sampling for further analysis. The selection considered both total test scores and the diversity of students' response patterns.

3. Research Instruments

The research instrument consists of an essay test. Essay tests are used to collect data on students' mathematical critical thinking abilities and mathematical communication skills in solving statistical problems. Critical thinking skills are assessed through five indicators based on Robert H. Ennis' standards: (1) analyzing problems, (2) formulating reasons, (3) proving them, (4) drawing conclusions, and (5) evaluating solutions. Mathematical communication skills are assessed through four indicators based on the National Council of Mathematics Teachers (NCTM) standards: (1) using mathematical terms, notations, or symbols to solve problems; (2) expressing or explaining mathematical ideas in the form of drawings or visual representations; (3) interpret mathematical drawings or models into ordinary verbal language; and (4) interpret notation and symbols in a mathematical context. These indicators are operationalized into assessment rubrics to ensure consistency in the assessment of student responses. To maintain the consistency

of scoring, all student answers are assessed using the same rubric, and a re-examination of the scoring results is carried out before the analysis is carried out.

Each student's response was assessed using an analytic rubric consisting of three components: information identification, solution process, and conclusion. Information identification and solution process were scored on a scale of 0–3 (or 0–5 for items requiring visual representations), while conclusions were scored on a scale of 0–2. The maximum scores were 77 for mathematical critical thinking and 80 for mathematical communication. The obtained scores were converted into percentages using the formula: final score = (obtained score/maximum score) × 100.

4. Test Instruments

Before being used in research, the instrument is validated by an expert lecturer in the field of mathematics education. Validation was carried out on aspects of indicator suitability, construct representation, readability, language clarity, and the suitability of the context of the question. The validation results show that the instrument is suitable for use after revisions are made to the editorial aspect and the suitability of the indicators. Furthermore, instrument tests were carried out, which included analysis of validity, differentiation, difficulty, and reliability. The results of the initial Instrument tests are shown in table 1 below.

Table 1. Recapitulation of the Validity of Critical Thinking Ability Instruments

Recapitulation of the Validity of Critical Thinking Ability Instruments							
No Question	**Validity**		**Differentiation**		**Difficulty Level**		**Remarks**
	Score	Criteria	Score	Criteria	Score	Criteria	
1.	0.680	Valid	0.497	Good	0.51	Medium	Used
2.	0.617	Valid	0.445	Good	0.59	Medium	Used
3.	0.830	Valid	0.707	Very good	0.39	Medium	Used
4.	0.890	Valid	0.797	Very good	0.28	Difficult	Used
5.	0.811	Valid	0.712	Very good	0.16	Difficult	Used
6.	-0.154	Invalid	-0.286	Very ugly	0.09	Difficult	Not used
7.	0.270	Invalid	0.228	Enough	0.02	Difficult	Not used
8.	0.519	Valid	0.500	Good	0.01	Difficult	Used
9.	0.519	Valid	0.500	Good	0.01	Difficult	Used
10.	-0.027	Invalid	-0.090	Very ugly	0.02	Difficult	Not used
Reliability of Critical Thinking Ability Instruments							
Crocbach's Alpha							0.727
Categories							Height

Based on table 1, seven of the ten items met the validity criteria and were retained for the study, while three items were excluded due to insufficient validity. The retained items generally demonstrated good to very good discrimination power and ranged from moderate to difficult levels of difficulty, indicating their suitability for assessing higher order critical thinking skills in statistics. The instrument also showed good internal consistency, with a Cronbach's Alpha coefficient of 0.727. Overall, the results indicate that the selected items met the required validity, discrimination, difficulty, and reliability criteria and were therefore appropriate for data collection.

Table 2. Recapitulation of the Validity of Mathematical Communication Instruments

Recapitulation of the Validity of Mathematical Communication Instruments							
No Question	**Validity**		**Differentiation**		**Difficulty Level**		**Remarks**
	Score	**Criteria**	**Score**	**Criteria**	**Score**	**Kriteia**	
1.	0.712	Valid	0.660	Good	0.65	Medium	Used
2.	0.987	Valid	0.911	Very good	0.50	Medium	Used
3.	0.821	Valid	0.756	Very good	0.50	Medium	Used
4.	0.858	Valid	0.821	Very good	0.52	Medium	Used
5.	0.847	Valid	0.783	Very good	0.49	Medium	Used
6.	0.871	Valid	0.830	Very good	0.41	Medium	Used
7.	0.730	Valid	0.656	Good	0.60	Medium	Used
8.	0.898	Valid	0.849	Very good	0.67	Medium	Used
Instrument Reliability, Mathematical Communication Ability							
Crocbach's Alpha							0.72
Categories							Height

Based on table 2, all eight items met the validity criteria and were retained for the study. The items generally demonstrated good to very good discrimination power and moderate levels of difficulty, indicating their suitability for assessing students' mathematical communication skills in statistics. The instrument also showed good internal consistency, with a Cronbach's Alpha coefficient of 0.792. Overall, the results indicate that all items met the required validity, discrimination, difficulty, and reliability criteria and were therefore appropriate for data collection.

5. Data Collection Techniques

Data collection is carried out through several stages. The first stage is to give tests to all students to obtain initial data on mathematical critical thinking

skills and mathematical communication skills. Students' answers are assessed using an assessment rubric based on indicators of each ability. The score for each answer is summed and then converted to a percentage using the formula: percentage = (score obtained/maximum score) × 100%.

The percentage of student achievement is further classified into ability categories to facilitate the interpretation of student achievement levels. The ability categorization criteria used in this study were adapted from Maya (2011). The categorization is used to describe the level of students' ability as well as determine representative subjects that represent the high, medium, and low categories to be analyzed in more depth. The results of the categorization of students' abilities are presented in table 3.

Table 3. Mathematical Ability Categorization Criteria

Percentage	Categories
$> 70\%$	Height
$55\% - 70\%$	Medium
$\leq 55\%$	Low

Based on the results of the categorization, three students representing the high, medium, and low categories were selected to analyze their answers.

6. Data Analysis and Validity

Data analysis was carried out in two stages. The first stage is a descriptive quantitative analysis through the calculation of scores and the percentage of student achievement on each indicator to describe the distribution of abilities and determine representative subjects. The second stage is qualitative analysis through data reduction, data presentation, interpretation, and drawing conclusions based on written answers from students. Data integration is carried out at the interpretation stage by using quantitative results as the basis for subject selection and qualitative results as the basis for explaining the characteristics of students' abilities.

In qualitative research, the use of clear and systematic validation procedures is important for enhancing the credibility and trustworthiness of research findings (Creswell & Poth, 2018). Therefore, expert validation, instrument testing, and consistent scoring procedures were employed in this study to support data quality.

RESULTS AND DISCUSSION

The results of the study in general show that both the mathematical critical thinking skills and mathematical communication skills of grade VIII students in solving statistical problems remain in the low category. Of the 24 students, 15 were in the low critical thinking category. Meanwhile, mathematical communication skills were dominated by the low category, with 23 students in the category and only 1 student in the medium category. This condition reflects that students have not been able to optimally integrate logical thinking processes with the ability to express mathematical reasoning in the context of statistical problem solving. These findings are consistent with research showing that high-level thinking skills are still a challenge in mathematics education because learning often focuses on mastering procedures and algorithms, while the ability to reason, interpret results, and communicate mathematical ideas has not been optimally developed (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017).

Table 4. Profile of the Level of Critical Thinking and Mathematical Communication Skills in Class VIII G

No.	Critical Thinking	Number of Students	Mathematics Communication	Number of Students
1.	High	5	High	0
2.	Medium	4	Medium	1
3.	Low	15	Low	23
		Total		24

Based on this grouping, 3 students were selected as research subjects for further analysis. The selected research subject is S17, a subject with a high category. S1 is a subject with a medium category, and S20 is a subject with a low category.

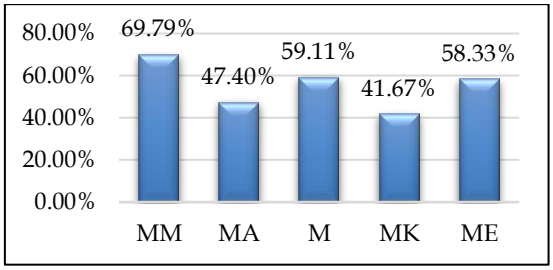


Figure 1. Percentage of Each Indicator: Mathematical Critical Thinking Ability Indicators used: MM = Analyze Problems; MA = Structuring Reason; M = Prove; MK = Draw Conclusions; ME = Evaluate.

The problem analysis indicator obtained the highest percentage among all critical thinking indicators at 69.79%, indicating that most students have been able to identify the information provided and what is asked in the problem, as well as to determine the initial steps towards a solution. These results reflect the basic comprehension skills that students generally develop through repeated exposure to structured problem types. The average overall mathematical critical thinking ability of students was obtained at 55.26%, which was calculated from the average percentage of the five indicators. Based on the categorization criteria adapted from Maya (2011), this average is in the low category (≤55%).

The proof indicator obtained a percentage of 59.11%, while the evaluation indicator reached 58.33%. These results show that students are quite capable of providing evidence for their solutions and double-checking their answers. However, a large number of responses still contain errors in the calculation process and inaccuracies in mathematical reasoning. Difficulties at the evaluation stage are consistent with research conducted by Anggraini, Siagian, and Agustinsa (2022), which found that students tend to struggle with evaluations because they are not used to systematically verifying the correctness of their procedures and solutions. Evaluation requires a metacognitive orientation, the ability to step back from solutions and assess their validity, which is rarely developed through conventional mathematics instruction. The reasoning structuring indicator obtained a percentage of only 47.40%, which reflects a relatively weak ability to build logical arguments or justify the chosen solution approach. Based on the analysis of students' answers, most students only write down the calculation step without an explanation of why the step was chosen. Low category students such as S20 do not even write down reasons at all, while S1 only gives descriptive reasons. This condition is likely to occur because students are more accustomed to learning that emphasizes the final result of calculations, so the ability to build logical arguments has not been adequately trained. This is in line with Andriani, Subanji, and As'ari (2021), which states that the ability to build mathematical reasoning requires continuous practice through learning activities that emphasize discussion, argumentation, and reflective reasoning. Without intentional instructional scaffolding in these areas, students are unlikely to develop the argumentative skills necessary for high-level math performance.

The conclusion indicator obtained the lowest percentage, which was 41.67%, representing the most fundamental weakness in students' critical thinking performance. Most students do not write a systematic conclusion that

is contextually aligned with the problem they have solved. This low inference ability suggests that students continue to face obstacles in synthesizing the results of their problem solving process into coherent closing statements. Hidayat and Noer (2021) demonstrated that the ability to draw mathematical conclusions consistently is one of the lowest achievement indicators of critical thinking, as students tend to focus on computational processes rather than formulating final interpretive assessments. These findings are further supported by Lithner (2017), who observed that students were more accustomed to following calculation procedures than to constructing deductive reasoning independently.

Analysis of students' answer patterns reveals some consistent errors during statistical problem solving. Most students tend to write the final calculation results directly without including systematic problem-solving steps, making it impossible to clearly identify the reasoning process used. Students also have difficulty reading and interpreting the data presented in tables and graphs, leading to errors in choosing the right solution strategy. In addition, many students do not write down the conclusion that is in accordance with the context of the problem, even though the calculations made are correct. This condition shows that students' ability to interpret the results and convey the mathematical meaning of the completed results still needs to be improved. The findings indicate that mathematical analysis skills have not been developed optimally, even though these skills play an important role in understanding and explaining the results of problem solving (Izzati, 2017).

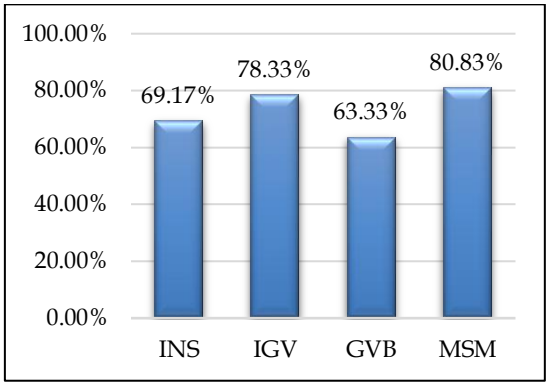


Figure 2. Percentage of Each Indicator of Mathematical Communication Ability

Indicators: INS = Notation and Interpretation of Symbols; IGV = Visual Image Idea; GVB = Ordinary Verbal Image; MSM = Using mathematical symbols

The indicator of the use of mathematical symbols (MSM) reached the highest percentage of 80.83%. These results show that most students can use mathematical terms, notations, and symbols appropriately in solving the given problems. This ability can be seen from the accuracy of students in writing mathematical symbols and performing calculation procedures according to the concepts used. Meanwhile, the average overall mathematical communication skills of students were 72.92%, which was obtained from the average percentage of the fourth mathematical communication indicator.

The visual image idea indicator (IGV) obtained a percentage of 78.33%. These results show that students are quite capable of presenting mathematical information in visual form, such as diagrams or graphs. Students can make visual representations that accurately match the data provided so that mathematical information can be presented in a clearer, more structured, and easier to understand manner.

The symbol notation and interpretation indicator (INS) reached a percentage of 69.17%. These results show that most students are able to express the information contained in the problem in the form of appropriate mathematical symbols or notations. However, there are still some students who have difficulty determining the right symbol to represent the information provided.

Meanwhile, the ordinary verbal image indicator (GVB) obtained the lowest percentage at 63.33%. These results show that students still have difficulty in interpreting the information presented through diagrams or mathematical models into the form of verbal language. Some students can identify the information contained in the diagram, but have not been able to explain the information completely and systematically. This finding is in line with Pratiwi, Putri, Zulkardi, and Hiltrimartin (2025) who stated that students still have difficulty communicating mathematical ideas and reasoning clearly through spoken and written language.

To obtain a deeper picture of students' critical thinking and mathematical communication skills, three representative subjects representing the high, medium, and low categories were selected based on the results of the critical thinking ability test. The subjects are S17 (high category), S1 (medium category), and S20 (low category). The selection of this subject aims to describe the characteristics of students' answers in each ability category.

Table 5. Characteristics of Students' Critical Thinking and Mathematical Communication Skills in Solving Statistical Problems

Student Code	Categories	Findings of Critical Thinking Ability	Findings of Mathematical Communication Ability	Interpretation
S17	Height	Able to analyze information, provide logical reasons, and evaluate the results of the settlement, even though they have not used formal statistical concepts consistently.	Able to use mathematical symbols appropriately, represent information, and explain ideas verbally and visually.	Demonstrate a good integration between critical thinking processes and mathematical communication. The ability to analyze and evaluate supports the ability to explain ideas systematically.
S1	Medium	Able to understand the problem and determine basic solutions, but the reasons and conclusions given are still descriptive.	Able to read diagrams and present data in visual form, but the interpretation given is not yet in-depth.	Sufficient critical thinking skills, followed by good communication skills, but neither has shown strong arguments.
S20	Low	Tends to focus on calculation procedures and final results without adequate proof or evaluation.	Able to use symbols and perform calculations, but not able to explain the reason or meaning of the results obtained.	The limitations of critical thinking have implications for incomplete and less reflective mathematical communication.

Based on table 5, there are differences in the characteristics of mathematical critical thinking and mathematical communication skills among each category of students. High category students (S17) showed a relatively better ability to integrate critical thinking processes with mathematical communication compared to middle and low category students. S17 can analyze the information provided, build logical reasoning, and evaluate the results obtained. At the same time, these students can use mathematical symbols correctly, represent information in various forms, and explain mathematical ideas both verbally and visually. These findings suggest that good critical thinking skills tend to support students' ability to communicate mathematical ideas more clearly and systematically. According to the National Council of Teachers of Mathematics (2000), mathematical communication is an essential process that allows students to organize and refine their mathematical thinking through representation, reasoning, and logical explanation.

Meanwhile, students in the secondary category (S1) show critical thinking and mathematical communication skills at a fairly adequate level, although they have not developed optimally. Students can understand the problem and determine the basic solution, but the reasons given are still descriptive and do not show a deep argument. In terms of mathematical communication, students can read diagrams and present information visually, but their interpretation of data is still limited to stating visible facts without connecting them to broader meanings. This situation shows that critical thinking and mathematical communication skills develop at the same time, but the qualities of both are still in the transition phase from procedural to conceptual understanding. Paul and Elder (2020) argue that critical thinking involves not only the ability to get the right answers but also the ability to give rational reasons, reflect, and evaluate the quality of the arguments made.

In contrast to the previous two subjects, students in the low category (S20) tend to show abilities that still focus on procedural aspects. They can do calculations and get the right final result, but they have not been able to provide evidence, evaluation, or explanation of the concepts of the steps taken. In terms of mathematical communication, students can use mathematical symbols and do simple calculations, but they cannot explain the reason or meaning behind the results. This suggests that the mathematical communication shown is still mechanical and does not reflect a deep understanding of the statistical concepts used. According to Lithner (2017), students who are accustomed to relying on procedural reasoning tend to focus on the application of algorithms without

developing mathematical arguments that support their problem-solving process.

Overall, the results of the analysis show a tendency to have a positive relationship between students' mathematical critical thinking skills and their mathematical communication skills in solving statistical problems. Students who have better critical thinking skills tend to be able to convey mathematical ideas in a more complete, systematic, and meaningful way. On the other hand, limitations in critical thinking skills are often followed by less in depth mathematical communication and focus only on the final result. These findings support the view that critical thinking skills and mathematical communication are two complementary competencies in the process of solving mathematical problems. The ability to analyze, reason, evaluate, and draw conclusions constitutes a fundamental aspect of mathematical thinking that underpins students' capacity to communicate mathematical ideas effectively through symbolic, visual, and verbal representations. By engaging in these higher-order cognitive processes, students can interpret mathematical information, construct logical arguments, evaluate the validity of solution strategies, and formulate well-supported conclusions. Consequently, they are able to communicate their mathematical reasoning more clearly, coherently, and accurately, thereby promoting both conceptual understanding and effective mathematical communication.

Based on the analysis of the characteristics of students' answers in the high, medium, and low categories, it was found that mathematical critical thinking and mathematical communication do not emerge as separate skills, but rather are interrelated in the process of solving statistical problems. The difference in students' critical thinking abilities seems to be in line with the quality of mathematical communication they demonstrate. Students who are able to better analyze information, provide reasons, evaluate results, and draw conclusions also tend to show better abilities in representing, explaining, and communicating mathematical ideas. On the other hand, students who still focus on calculation procedures without sufficient arguments and reflection tend to show incomplete mathematical communication. Therefore, to gain a deeper understanding of the pattern of relationships between these two abilities, a cross-case analysis was conducted on three representative subjects representing the high, medium, and low categories. The results of the analysis of the relationship between mathematical critical thinking and students' mathematical communication skills in solving statistical problems are presented in table 6.

Table 6. Analysis: The relationship Between Mathematical Critical Thinking Skills and Students' Mathematical Communication skills

Student Categories	Capabilities Critical Thinking	Mathematical Communication Skills	Visible linkages
S17	Able to analyze information, provide reasons, and evaluate the results of completions.	Able to use mathematical symbols, represent information, and explain ideas verbally and visually.	Good analytical and evaluation skills support students' ability to communicate mathematical ideas clearly, systematically, and using appropriate representations.
S1	Able to understand the problem and determine solutions, but the reasons and conclusions are still descriptive.	Able to read diagrams and present visual information, but interpretation is still limited.	Sufficient critical thinking skills produce good mathematical communication, but the arguments and interpretations conveyed are not in-depth.
S20	Focuses on calculation procedures and final results without adequate evaluation or proof.	Able to use mathematical symbols and perform calculations, but not yet able to explain the reason and meaning of the results.	The limitations in critical thinking cause the mathematical communication displayed to be only procedural and less reflective.

Based on table 6, there seems to be a consistent relationship between students' mathematical critical thinking abilities and their mathematical communication skills when solving statistical problems. Students who belong to the high category show better ability to analyze information, give reasons, and evaluate problem-solving results. This ability goes hand in hand with more complete mathematical communication, characterized by the use of appropriate mathematical symbols, the ability to represent data, and the ability to explain mathematical ideas systematically. These findings suggest that good critical

thinking helps students organize and communicate their thoughts more clearly when solving problems.

In the medium category, critical thinking and mathematical communication skills seem to develop at the same time, although they have not yet reached optimal depth. Students can understand the problem and present relevant information, but the reasoning, interpretation, and conclusions they provide are still descriptive. This shows that mathematical communication skills are influenced not only by mastery of mathematical representations but also by students' ability to reason and reflect on the information they get. In other words, limitations in the critical thinking process also limit the quality of mathematical communication displayed.

Meanwhile, students in the lower category tend to focus on calculation procedures and final results without providing adequate evidence or evaluation. This pattern affects mathematical communication, so it is limited to the use of symbols and the presentation of results without an explanation of the concepts that support it. These findings suggest that effective mathematical communication requires critical thinking skills as a basis for explaining reason, interpreting results, and constructing logical mathematical arguments. Recent studies have also shown that mathematical communication skills are among the high-level thinking skills that play an important role in supporting the resolution of complex mathematical problems (Erviana, Noer, & Triana, 2025).

Overall, the analysis The Relationship Between Mathematical Critical Thinking Skills and Students' Mathematical Communication Skills shows that the higher a student's mathematical critical thinking ability, the better the quality of mathematical communication they demonstrate. The ability to analyze, reason, evaluate, and draw conclusions also supports students' ability to represent and convey mathematical ideas in symbolic, visual, or verbal form. On the other hand, limitations in critical thinking tend to result in mathematical communication that is procedural and lacks depth. These findings confirm the view that these two abilities support each other in the process of solving statistical problems and need to be developed together in mathematics learning (Agusman, Purwanto, & Rahardi, 2025).

CONCLUSION

This study shows that mathematical critical thinking skills and mathematical communication skills are two interrelated abilities in the statistical problem solving process. The quality of student problem solving is

not only determined by the ability to obtain the correct answers, but also by the ability to analyze information, provide logical reasons, evaluate solutions, and communicate mathematical ideas clearly and meaningfully. The research findings indicate that students who are able to demonstrate better reasoning tend to have more systematic mathematical communication skills, while students who have difficulty building arguments and evaluating information tend to display communication that is procedural and less reflective.

The difficulties experienced by students are mainly seen in the ability to draw conclusions and interpret results according to the context of the problem. This condition shows that some students are still accustomed to using calculation procedures without linking the results obtained to the underlying mathematical meaning. Thus, the main challenge in learning statistics lies not only in mastering concepts and procedures, but also in developing the ability to reason, reflect, and communicate the results of thinking mathematically.

These findings imply that statistical learning needs to be designed to encourage students to explain reasoning, evaluate solution strategies, interpret results, and communicate mathematical ideas through a variety of representations. The development of mathematical critical thinking skills and mathematical communication needs to be carried out in an integrated manner because both play an important role in supporting meaningful problem solving. This research was limited to a relatively small number of subjects and was only conducted on statistical materials. Therefore, further research may involve a wider number of participants, different math materials, or use a research design that allows for a more in-depth exploration of the relationship between the two abilities.

ACKNOWLEDGMENT

The authors would like to express their sincere gratitude to the Principal and mathematics teachers of SMP Negeri 15 Cirebon City for granting permission and facilitating the data collection process. The authors also gratefully acknowledge all Grade VIII students who participated in this study as research subjects. Their cooperation and participation contributed significantly to the successful completion of this research.

DECLARATIONS

Author Contribution	:	NI and DAS : The Conception and design of the study, Instrument Development, Data collection, Data analysis, Interpretation of findings, manuscript drafting, critical revision of the manuscript, and approval of the final version for publication.
Funding Statement	:	The authors received no financial support for the research, authorship, and/or publication of this article.
Conflict of Interest	:	The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Additional Information	:	Additional information is available from the corresponding author upon reasonable request.

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