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An Analysis of the Queuing System in the Issuance of Identity Cards (KTP) at the Department of Population and Civil Registration, Barru Regency

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ABSTRACT
This study examines the queuing system for e-KTP recording services at the Population and Civil Registration Office of Barru Regency. Using a Multi-Channel Single-Phase queue model, the objective is to optimize the queuing process and evaluate system performance. The Multi-Channel Single-Phase model is applied because the service involves a single type of process with multiple service facilities. The findings show that, over a five-day observation period, the system achieved a performance efficiency level with a steady-state measure of $\rho < 1$, indicating that steady-state conditions were satisfied. Thus, the results suggest that the e-KTP recording services at the Population and Civil Registration Office of Barru Regency operate optimally.

KEYWORDS
Queueing Theory, Queueing System, Service, Multi-Channel Single-Phase.

1. INTRODUCTION

A queue arises when the demand for a service exceeds the capacity of the service provider. In many public facilities, queues occur as individuals arrive, join the line, wait, and proceed until the service process is completed. Long waiting times can be minimized through more efficient time management. Key variables such as arrival time, service time, and completion time can be analyzed using queuing theory. This approach is particularly useful for addressing excessive queues, such as those observed in the issuance of electronic identity cards (KTP-el) at the Department of Population and Civil Registration (Disdukcapil). As the administrative office responsible for issuing identity cards to all registered residents of Barru Regency, Disdukcapil frequently encounters high service demand.

The queuing process at Disdukcapil Barru consists of several stages. Service seekers must first queue to obtain a number for the designated service counter. There are three service counters: Counter 1 for KTP-el recording, Counter 2 for printing, and Counter 3 for collection. Among these, Counter 1 often experiences congestion due to the high number of applicants. The queuing system at Counter 1 follows a Multi-Channel Single-Phase model, where two service officers serve customers through a single queue line.

The methodology of this study involved several stages: data collection, data analysis, and conclusion drawing. Direct observation was applied to gather data on the queuing system. Afterward, the arrival distribution and service times were determined and used to evaluate the system's performance by applying formulas aligned with the chosen queuing model.

2. LITERATURE REVIEW

2.1 Queuing Theory

Queuing theory is the systematic study of queues or waiting lines. A queue occurs when the demand for a service exceeds the available service capacity, forming lines of waiting individuals. Appropriate decisions must be made regarding the number of servers or facilities, even though the arrival of individuals in need of service is uncertain and the time required to serve each customer is unpredictable [1].

In a queuing system, there may be one or more service providers available to handle various customer arrivals. When a newly arrived customer finds that all service providers are busy, they must join the queue before receiving service. Conversely, if a customer arrives when a server is idle, they can be served immediately without waiting [2].

2.2 Basic Elements in A Queuing System

The main elements of a queuing system are as follows [3]:

a Input Source (Population)

The input source refers to the pool of individuals from which customers arrive to request services. The population may be finite or infinite, depending on its size. A finite population occurs when the number of potential customers is relatively small and countable. Conversely, when the number is very large and uncertain, the population is considered infinite. For example, patients arriving at an emergency clinic represent an infinite population.

b Arrival Pattern

The arrival pattern describes the manner in which individuals enter the system. Arrival rates may be constant or random, depending on the service context.

c. Queue Discipline Queue discipline refers to the rules or principles governing the order in which customers are served. Common types of service disciplines include:

1. *First Come, First Served (FCFS)* or *First In, First Out (FIFO)* — customers are served in the exact order of their arrival.
2. *Last Come, First Served (LCFS)* or *Last In, First Out (LIFO)* — the most recent customer to arrive is served first.
3. *Service in Random Order (SIRO)* — customers are selected for service randomly, regardless of their arrival time.
4. *Priority Service (PS)* — certain customers are served first based on urgency or priority needs [4].

d. Queue Length

Queue length refers to the maximum number of customers that a system can accommodate while waiting for service. Some queuing systems allow for a large or unlimited number of customers (infinite queue length), while others have restricted capacity (finite queue length), limiting the number of individuals that can wait.

e. Service Rate

The service rate is the speed at which customers are served, typically expressed as the number of customers served per unit of time. It depends on the efficiency of the system and may be constant or random.

f. Service Time Distribution

Service time distribution describes the variation in service duration across customers. It may be constant—where every customer requires the same amount of time—or random, following a probability distribution based on empirical observations.

g. Exit

After receiving service, customers leave the system. They may either return to the original population (with some probability of re-entry) or move to another system requiring fewer or different services.

2.3 Queue Structures

There are four fundamental models of queuing systems [5]:

a. *Single-Channel, Single-Phase*

This is the simplest model. A single channel indicates that there is only one service facility, while a single phase means the customer undergoes only one service activity. After receiving service, the customer immediately exits the system.

b. *Single-Channel, Multi-Phase*

In this model, there is only one service facility, but customers must go through two or more service stages sequentially before completing the process.

c. *Multi-Channel, Single-Phase*

This model involves two or more service facilities operating in parallel, all serving customers from a single queue. Each customer receives service in only one phase.

d. *Multi-Channel, Multi-Phase*

This is the most complex model, where multiple types of services exist and each service type is provided by more than one facility. Each stage of the process may have multiple servers, allowing several customers to be served simultaneously.

2.4 Poisson and Exponential Distributions

a. **Poisson Distribution**

The Poisson distribution is a discrete probability distribution that describes the likelihood of a given number of arrivals within a specified time period, assuming that the average arrival rate is known and arrivals occur randomly and independently. If the expected number of events in an interval is λ, then the probability of observing x arrivals is given by:

$$p(x) = \begin{cases} \frac{e^{-\mu}\mu^{x}}{x!} & x = 0, 1, 2, 3, \ldots \end{cases} \tag{1}$$

where μ is the average number of experimental outcomes that occur in a stated time interval and $e = 2.71828\ldots$

b. **Exponential Distribution**

The exponential distribution, on the other hand, is commonly used to model service times in queuing systems where service durations are assumed to be random. In this case, the service time for a customer is independent of both the service time of previous customers and the number of customers currently waiting in the system.

A continuous random variable X has an exponential distribution over its parameters if its probability density function is as follows:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & for\, x > 0,\ \lambda > 0 \\ 0, & for\ other\ values\ of\ x \end{cases} \tag{2}$$

where x can express the time required to achieve an achievement and λ is the average number of successes in one unit of time interval [6].

2.5 Queuing Models

Queuing models can generally be expressed in the following notation:

$$(a/b/c) : (d/e/f)$$

where

a = Arrival distribution (number of arrivals per unit time),

b = Service time distribution,

c = Number of parallel servers ($c = 1, 2, 3, \ldots, \infty$),

d = Service discipline,

e = Maximum number of customers allowed in the system,

f = Population size.

Information:

1. Codes for a and b:
   a. M: Poisson distribution for arrivals and exponential distribution for service times (inter-arrival times follow an exponential distribution or arrivals follow a Poisson distribution).
   b. D: Deterministic (constant) inter-arrival or service times.
   c. G: General distribution for service times.
   d. E_k: Erlang or gamma distribution for inter-arrival and service times.
2. Codes for c: a positive integer representing the number of parallel servers.
3. Codes for d (General Service Discipline, GD):
   a. FIFO (First In First Out) or FCFS (First Come First Served),
   b. LIFO (Last In First Out) or LCFS (Last Come First Served),
   c. SIRO (Service In Random Order).
4. Codes for e and f:
   a. N (to indicate a limited number),
   b. ∞ (to indicate unlimited number of customers or population size)[7].

For example, consider the notation:

$(M/D/5/N/\infty)$

This means that arrivals follow a Poisson distribution, service times are constant, there are five servers, the maximum number of customers allowed in the system is N, and the source population is infinite.

a. Model $(M/M.1) : (G/\infty/\infty)$

   The queueing model uses an exponential distribution for service times and a Poisson distribution for arrival patterns. The queueing model uses a single line or a single service number with no call source restrictions. The equation for this queueing model is as follows:

   1. Probability of zero customers in the system

$$P_0 = 1 - \frac{\lambda}{\mu} \tag{3}$$

2. Average number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} \tag{4}$$

3. Average number of customers in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{5}$$

4. Average waiting time in the system

$$W_s = \frac{1}{\mu - \lambda} = \frac{L_s}{\lambda} \tag{6}$$

5. Average waiting time in the queue

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \tag{7}$$

6. Probability of the system (server) being busy

$$\rho = \frac{\lambda}{\mu} \tag{8}$$

Where λ is average arrival and μ is average service rate [8].

b. Model $(M/M/C):(G/\infty/\infty)$ The service and arrival distributions follow a Poisson distribution, and the service rules are general in queuing models with multiple lines or multiple queues. The service rate must be higher in this model than the arrival rate. This model has unlimited call source capacity and no system constraints. The queuing equation for this model is as follows:

1. Probability of zero customers in the system

$$P_0 = \frac{1}{\left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu - \lambda}\right)} \tag{9}$$

2. Average number of customers in the system

$$L_s = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!(s\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu} \tag{10}$$

3. Average number of customers in the queue

$$L_q = L_s + \frac{\lambda}{\mu} \tag{11}$$

4. Average waiting time in the system

$$W_s = \frac{L_s}{\lambda} \tag{12}$$

5. Average waiting time in the queue

$$W_q = W_s + \frac{1}{\mu} \tag{13}$$

6. Probability of the system (server) being busy

$$P_w = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu - \lambda}\right) P_0 \tag{14}$$

where s is a number of channels (number of services)[9].

2.6 Steady-State Measure

A condition in which the properties of a system do not change (constant) is a Steady state. For example, μ is the average number of customers served per unit of time, λ is the average number of customers arriving at the service point per unit of time and where ρ can be defined as the ratio between the average number of customers arriving per unit of time (λ) and the average number of customers served per unit of time (μ). Where c is the number of service servers in a service facility or can be written as follows [10].

$$\rho = \frac{\lambda}{c\mu} \tag{15}$$

The steady state probability in a given system is $\lambda < \mu$, so it can be written as $\rho < 1$. If the average number of arriving customers does not exceed the average number of customers served, then the steady state condition is met [11].

2.7 Goodness of Fit Test

A distribution fit test is used to evaluate how well sample data drawn from an unknown population conforms to a specified probability model. This test is useful for assessing the extent to which the model approximates real-world conditions. The Kolmogorov–Smirnov test is one of the most commonly used methods for evaluating distributional fit [12].

1. Arrival Distribution Test
   Hypotheses:
   H_0 : Customer arrivals follow a poisson distribution
   H_1 : Customer arrivals do not follow a poisson distribution
   with significance level $\alpha = 5\%$ and test statistic

   $$D = Sup\,|S(n) - F_0(n)| \tag{16}$$

   where $S(n)$ is cumulative distribution of the sample and $F_0(n)$ is cumulative distribution of the hypothesized distribution.
   Decision rule: Reject H_0 at $\alpha = 5\%$ if $D > D^*(\alpha)$. The critical value $D^*(\alpha)$ is obtained from the Komogorov Smirnov test table.

2. Service Distribution Test
   Hypotheses:
   H_0 : Service times follow an exponential distribution.
   H_1 : Service times do not follow an exponential distribution.
   with significance level $\alpha = 5\%$, test statistic as **Equation** (16), and the same decision rule with arrival distribution test.

3. METHODOLOGY

The type of research used in this study is applied research with the type of data in this study is quantitative data or data in the form of numbers, including analysis, hypotheses and data collection techniques. The data source is primary data originating from customers who record their e-KTP at the Barru Regency Population and Civil Registration Office. The variables used in this study are λ = Average number of customer appearances per unit of time (minutes), μ = Average number of customer services per unit of time (minutes), ρ = Level of facility utilization (%), L_q= Average number of customers in the queue (people), L_s = Average customers in the system (people), W_q = Average time in the queue (minutes) and W_s = Average time in the system (minutes)

3.1 Analysis Procedure

The steps of the research procedure are as follows:

1. Collect information or data from observations

   (a) Customer arrival times

(b) Customer service start times

(c) Customer service completion times

(d) Service duration

2. Calculate the size of the stable state by using estimates to find the values of λ and μ.

3. Conduct distribution tests for each data or information. Service distribution tests and arrival distribution tests.

4. Perform calculations to determine the size of the system performance.

5. Decision making based on the results of research that has been conducted.

4. RESULT & DISCUSSION

4.1 Queuing System

At the Department of Population and Civil Registration (Disdukcapil) of Barru Regency, two dedicated servers are utilized to facilitate the processing of electronic identity cards (e-KTP) for local residents. The e-KTP recording procedure encompasses several biometric and personal data collection stages, including photo capture, digital signature, fingerprint and iris scanning, as well as data personalization. Disdukcapil Barru applies a Multi-Channel Single-Phase queuing model for the KTP-el recording

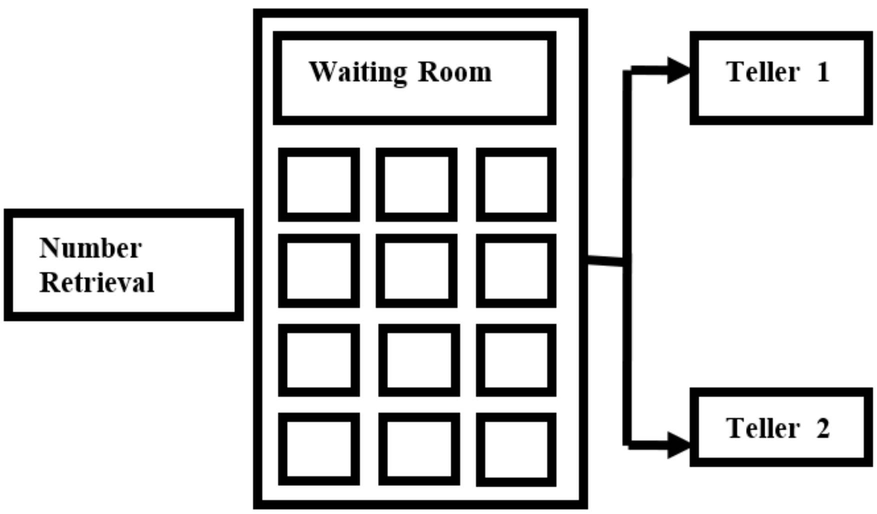

Figure 1. Queueing system diagram of Department of Population and Civil Registration (Disdukcapil) of Barru Regency

counter as depicted in the **Figure 1**. In this system, two servers can serve customers in parallel, but customers only go through one service phase. The service time for each customer varies randomly.The queuing discipline used is First Come First Served (FCFS), meaning that the first customer to arrive at the counter will receive a queue number and wait to be called in the same order for KTP-el recording services.

4.2 Steady State Measure

The results of the stage measurements can be seen in **Table 1**. Based on the results of the five-day study, the ρ value for the five days was $\rho < 1$. Therefore, the requirements for a steady-state condition have been met. This indicates that under steady-state conditions, the number of working hours of officers on duty at the client line for e-KTP recording is maximized. Therefore, the queuing system for e-KTP recording services at the Civil Registration Office (Disdukcapil) is considered optimal.

Table 1. Arrival rate (λ), service rate (μ), and utilization (ρ) per day

Day	λ	μ	ρ
Monday	10	5.41	0.0094
Tuesday	7	5.24	0.0070
Wednesday	8	5.053	0.0083
Thursday	6	4.72	0.0073
Friday	6	5.32	0.0058

Table 2. Poisson Distribution Test (Kolmogorov–Smirnov Test)

Test	D	p-value
Kolmogorov–Smirnov Test	0.85103	2.2×10^{-16}

4.3 Goodness of Fit Test

1. Arrival Distribution Test

   The results of the Poisson distribution test analysis can be seen in **Table 2**. Based on the results of the Kolmogorov–Smirnov test, the obtained D value is 0.85103 and the p-value is 2.2×10^{-16}. Since the D value (0.85103) is greater than the critical value $D^*(\alpha) = 0.0425$, and the p-value (2.2×10^{-16}) is less than the significance level $\alpha = 0.05$, the null hypothesis (H_0) is rejected. This indicates that the data on the number of customer arrivals do not follow a Poisson distribution. Consequently, the arrival process is assumed to follow a general (non-Poisson) distribution.

2. Service Ditribution Test

Table 3. Exponential Distribution Test (Kolmogorov–Smirnov Test)

Test	D	p-value
Kolmogorov–Smirnov Test	0.99437	2.2×10^{-16}

The results of the exponential distribution test analysis are presented in **Table 3**. As shown in the table, the Kolmogorov–Smirnov test yields a D value of 0.99437 and a p-value of 2.2×10^{-16}. Since the D value (0.99437) is greater than the critical value $D^*(\alpha) = 0.0497$, and the p-value (2.2×10^{-16}) is less than the significance level $\alpha = 0.05$, the null hypothesis (H_0) is rejected. This indicates that the customer service time data do not follow an exponential distribution. Consequently, the service time distribution is assumed to follow a general (non-exponential) form.

4.4 System Performance Measures

The service time pattern follows an exponential distribution. Meanwhile, the customer arrival rate is independent of time, meaning it's infinite, so it's random. Furthermore, the service discipline used is First Come First Served (FCFS), where customers who arrive first are served first at the counter. Based on **Table 4**, the average waiting time for the e-KTP recording

Table 4. Poisson Distribution Test Results

Day	W_q	L_q	L_s	W_s
Monday	1.571	0.160	0.0015	0.0147
Tuesday	2.262	0.167	0.0012	0.0162
Wednesday	2.010	0.167	0.0014	0.0169
Thursday	2.435	0.168	0.0012	0.0173
Friday	2.657	0.165	0.00096	0.0155

service at the Department of Population and Civil Registration of Barru Regency on Monday was 1.571 minutes. The average number of customers in the queue and in the system was approximately one customer every 10 minutes, with an average service time of 0.0147 minutes per customer. On Tuesday, the average waiting time was 2.262 minutes, with an average of one customer in the queue and system every 10 minutes, and an average service time of 0.0149 minutes.

On Wednesday, the waiting time for e-KTP recording at the Barru Regency Population and Civil Registration Office was 2.010 minutes, with an average number of customers in the queue and system of one customer per 10 minutes, and an average waiting time of 0.0169 minutes. On Thursday, the waiting time for e-KTP recording at the Barru Regency Population and Civil Registration Office was 2.435 minutes, with an average number of customers in the queue and system of one customer per 10 minutes, and an average waiting time of 0.0173 minutes. On Friday, the waiting time for e-KTP recording at the Barru Regency Population and Civil Registration Office was 2.657 minutes, with an average number of customers in the queue and system of one customer per 10 minutes, and an average waiting time of 0.0155

5. CONCLUSION

The conclusion of this study, based on the results of the description of the queuing system work with the model $(G/G/2):(G/\infty/\infty)$. It can be seen that the queuing work system uses a Multi Channel Single Phase queuing system. By looking at the level of service intensity $\rho < 1$, it can be concluded that the e-KTP registration service at the Department of Population and Civil Registration of Barru Regency operates optimally.

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