Jurnal Matematika dan Statistika serta Aplikasinya, Vol. 13, No. 2, 116-122, 2025
DOI:https://doi.org/10.24252/msa.v13i2.56235

Chromatic Number of the Corona Product of Complete Graph and Star Graph

Mustika Asmarani[1], Kartika Yulianti[2]*, Endang Cahya Mulyaning Asih[3]
[1,2,3] *Universitas Pendidikan Indonesia, Indonesia*
***Corresponding author**: kartika.yulianti@upi.edu

***Submission date**: 23 March 2025, **Revision**: 11 August 2025, **Accepted**: 03 November 2025

ABSTRACT
In this paper, we determine the chromatic number of the corona product of complete graph K_n and star graph $K_{1,m}$. Determination of the chromatic numbers is done by observing patterns, constructing conjectures, and proving them formally. We found that the chromatic number of the corona product of complete graph K_n and star graph $K_{1,m}$ is $\chi(K_n \odot K_{1,m}) = \begin{cases} 3, & n = 1,2 \\ n, & n = 3,4,\ldots,\ k \end{cases}$, and the chromatic number of the corona product of star graph $K_{1,m}$ and complete graph K_n is $\chi(K_{1,m} \odot K_n) = n+1,\ n = 1,2,\ldots,\ k$. Furthermore, using Matlab software, a program is created to visualize the coloring of the vertices based on the formula that has been obtained.

KEYWORDS
Coloring vertex, Chromatic number, Corona product, Complete graph, Star graph.

1. INTRODUCTION

A graph G can be formally defined as an ordered triple $(V(G), E(G), \psi_G)$ consisting of a nonempty set $(V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$, of edges, and ψ_G is an incidence function that maps each edge to an unordered pair of vertices within G [1], [2]. There are several graphs that have special characteristics, including complete graph, cyclic graph, regular graph, and bipartite graph. Complete graph (K_n) is a simple graph with n vertices, where each pair of vertices is joined by an edge [1], [3], [4]. A bipartite graph is a graph in which the vertices can be divided into two disjoint groups, X and Y, such that every edge connects a vertex in X to one in Y [1]. A complete bipartite graph $(K_{n,m})$ is a simple bipartite graph with bipartition (X,Y) in which each vertex of X is joined to each vertex of Y, where $|X| = n$ and $|Y| = m$ [1]. Specifically, complete bipartite graph $(K_{1,m})$ is named star graph [4].

Operations between two graphs can produce a new graph. Operations on the graph include Cartesian operation $(\times)$, join $(+)$, unioun $(\cup)$, comb $(\triangleright)$, and corona product $(\odot)$. The corona product of G and H, denoted $G \odot H$ is constructed by starting with a single copy of G and $|V(G)|$ copies of H, that is H_i with $i = 1,2,3,\ldots,|V(G)|$, and then making the i-th vertex of G adjacent to every vertex of the H_i [3], [5], [6].

One of the studies in graph theory is vertex coloring. A k-vertex coloring of G is an assignment of k colors to the vertices of G [1], [7]. The coloring is proper if no two distinct adjacent vertices have the same color. The chromatic number $\chi(G)$ of G is the minimum k for which G is k-colorable [1].

Study of the chromatic number of the corona product have been carried out by some researcher, includes Simanjuntak and Mulyono [8] that examine pattern chromatic number of the corona product of the cycle graph C_n and cubic graph Q_m, that are $\chi(C_n \odot Q_m)$ and $\chi(Q_m \odot C_n)$. Mohan, et al. [6] established the tight bound of the Behzad and Vizing conjecture regarding total coloring for the corona product of two graphs G and H, when H is a cycle, a complete graph or a bipartite graph. Meanwhile Ginting and Mulyono[5] studied the chromatic number of the corona product involving a star graph and a cycle. In contrast to the previous research, in this research, the chromatic number of the corona product of complete graph and star graph, that are

$\chi(K_n \odot K_{1,m})$ and $\chi(K_{1,m} \odot K_n)$.

By utilizing technological developments, to display the coloring of vertices in the graph resulting from corona product $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$, so in this paper, a program also be constructed to visualize vertex coloring using Matlab R2022a software.

2. LITERATURE REVIEW

In this chapter, we describe the definition of the corona product of two graphs and several theorems that will be used to prove the proposition of the chromatic number of the vertices of graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$. According to Harary & Frucht [3], and Mohan, et al.[6], the corona product of G and G is the graph $G \odot H$ obtained by taking one copy of G and $|V(G)|$ copies of G, that is H_i with $i = 1, 2, 3, \ldots, |V(G)|$, and making the i-th vertex of G adjacent to every vertex of the H_i.

Figure 1 is an example of the corona product of the star graph $K_{1,3}$ and complete graph K_3, denoted by $K_{1,3} \odot K_3$.

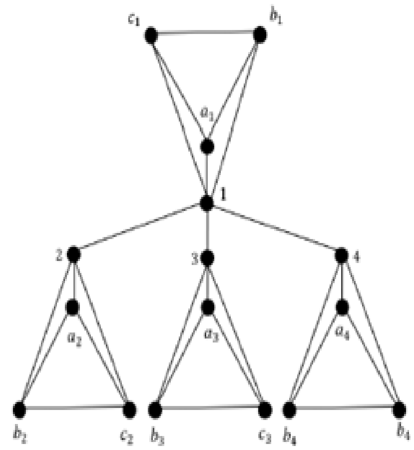

Figure 1. Corona product of star graph and complete graph $K_{1,3} \odot K_3$.

Theorem 2.1. If G is a complete graph with n vertices, then $\chi(G) = n$ [9]

Theorem 2.2. Let G is a connected graph. G is a bipartite graph if anly if $\chi(G) = 2$ [1].

Corollary 2.3. For star graph $K_{1,m}$, then $\chi(K_{1,m}) = 2$.

Proof. Star graph $K_{1,m}$ is a connected bipartite graph (X,Y) with $|X| = 1$ and $|Y| = m$. According to **Theorem 2.2**, then $\chi(K_{1,m}) = 2$. □

3. METHODOLOGY

The steps to obtain the chromatic number of graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$ are as follows:

1. Graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$ are figured for several m and n.
2. The structure of graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$ are studied by identify the number of edges, the number of vertices, and the maximum degree.
3. Doing vertex coloring, then the chromatic numbers of the graphs $K_n \odot K_{1,m}\ and K_{1,m} \odot K_n$ are obtained for several n and m.
4. Pattern of the chromatic simpul graf $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$ are observed.
5. Build a hypotesis for the chromatic numbers of the graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$.

6. Proof the hypothesis.

The vertex coloring visualization program for the graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$ is constructed by following steps:

1. Designing an algorithm for vertex coloring visualization program of graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$.
2. Create a Graphical User Interface (GUI) in Matlab as the front display when running the program.
3. Create a program script in Matlab based on the designed algorithm.

4. RESULT & DISCUSSION

4.1 The Chromatic Number of Graph $K_n \odot K_{1,m}$

Proposition 4.1. The chromatic number of graph $K_n \odot K_{1,m}$, $m = 1,2,\dots,p$ is

$$\chi(K_n \odot K_{1,m}) = \begin{cases} 3, & n = 1,2, \\ n, & n = 3,4,\dots,k. \end{cases}$$

Proof. The chromatic number of $K_1 \odot K_{1,m}$ is 3. The vertex coloring for graph $K_1 \odot K_{1,m}$ can be stated as the function

$$f(v) = \begin{cases} w_1, & v = a, \\ w_2, & v = b, \\ w_3, & v = c_{1j},\ j = 1,2,\dots,m, \end{cases}$$

where a is a vertex of K_1, b is a center vertex of $K_{1,m}$, and c_{1j} are pendant vertices in $K_{1,m}$.

The chromatic number of $K_2 \odot K_{1,m}$ is 3, with the vertex coloring function defined by

$$f(v) = \begin{cases} w_1, & v \in \{a_1, b_2\}, \\ w_2, & v \in \{b_1, a_2\}, \\ w_3, & v = c_{ij},\ i = 1,2 \text{ and } j = 1,2,\dots,m, \end{cases}$$

where a_i are vertices of K_2, b_i are center vertices of $K_{1,m}$, and c_{ij} are pendant vertices at $K_{1,m}$.

Next, it will be shown that $\chi(K_n \odot K_{1,m}) = n$ for $n = 3,4,\dots,k$. Let the vertex set of K_n be $A = \{a_1, a_2, \dots, a_n\}$. Steps to color vertices of $K_n \odot K_{1,m}$ are as follows:

1. Coloring the vertices in K_n is done first. Obtained $\chi(K_n) = n$ and let the color set is $W = \{w_1, w_2, \dots, w_n\}$ where w_i are color for vertex a_i.
2. Next, give color to the vertices in $K_{1,m}$, where $\chi(K_{1,m}) = 2$. Since each vertex of the graph $K_{1,m}$ adajcent with a vertex in K_n, then there are $n-1$ colors in W that can be chosen as two colors to color the graph $K_{1,m}$.
3. Vertices in $K_{1,m}$ that adjacent with a_1 in K_n and vertices in $K_{1,m}$ that adjacent with a_2 in K_n, colored first using the same colors as in $K_2 \odot K_{1,m}$.
4. Then, coloring is done for the vertices in $K_{1,m}$ which adjacent with a_3 in K_n. Since every vertex in $K_{1,m}$ adjacent with the vertex a_3 in K_n, then color w_3 can not be used to color vertices in $K_{1,m}$. So, it was chosen w_2 to color the center vertex (denoted by b_3) in $K_{1,m}$ and the color w_1 to color pendant vertices (denoted by $c_{3,j}$, $1 \le j \le m$) in $K_{1,m}$.
5. Next, coloring is done for the vertices in $K_{1,m}$ that are adjacent to vertex a_4 in K_n. Since every vertex in $K_{1,m}$ adjacent with the vertex a_4 in K_n, then the color w_4 can not be used to color the vertices in $K_{1,m}$. So, it was chosen w_2 to color the center vertex (denoted by b_4) in $K_{1,m}$ and w_3 to color pendantvertices (denoted by $c_{4,j}$, $1 \le j \le m$) in $K_{1,m}$.

6. Giving color to the vertices in $K_{1,m}$ that adjacent to vertices a_5,a_6, $\ldots$, a_n *in* K_n is done by similar ways to color $K_{1,m}$ which adjacent to a_4 in K_n.

So, the smallest number of colors required to color the vertices of $K_n \odot K_{1,m}$, $n \geq 3$ is n, that is $\{w_1,\ w_2,\ \ldots,\ w_n\}$. The vertex coloring for graph $K_n \odot K_{1,m}$ can be stated by the function:

$$f(v) = \begin{cases} w_1, & v = \{a_1, b_2, c_{3j} \mid j = 1,2,\ldots,m\}, \\ w_2, & v = \{a_2, b_i \mid i = 1,2,\ldots,n\} \setminus \{b_2\}, \\ w_3, & v = \{a_3, c_{ij} \mid i = 1,2,\ldots,n;\ i \neq 3,\ j = 1,2,\ldots,m\}, \\ w_v, & v = \{a_n \mid n = 4,5,\ldots,n\}, \end{cases}$$

where a_i are vertices at K_n, b_i are center vertices at $K_{1,m}$, and c_{ij} are pendant vertices at $K_{1,m}$.

As an example, **Figure 2** illustrates the vertex coloring of graph $K_6 \odot K_{1,3}$. Vertices at K_6 can be colored by six colors, namely:w_1, w_2, w_3, w_4, w_5, and w_6. Then, for the six copies of $K_{1,3}$, each can be colored by two colors. For graph $K_{1,3}$ adjacent to the vertex with color w_1 in K_6, we can assign color w_2 for the center vertex and color w_3 for the pendant vertices. For graph $K_{1,3}$ adjacent to the vertex with color w_2 in K_6, the center vertex can be colored by w_1 and the pendant vertices by w_3. For graph $K_{1,3}$ adjacent to the vertex with color w_3 in K_6, the center vertex can be colored by w_2 and the pendant vertices by w_1. For graph $K_{1,3}$ adjacent to the vertices with colors w_4, w_5, or w_6 in K_6, the center vertex can be colored by w_2 and the pendant vertices by w_3. □

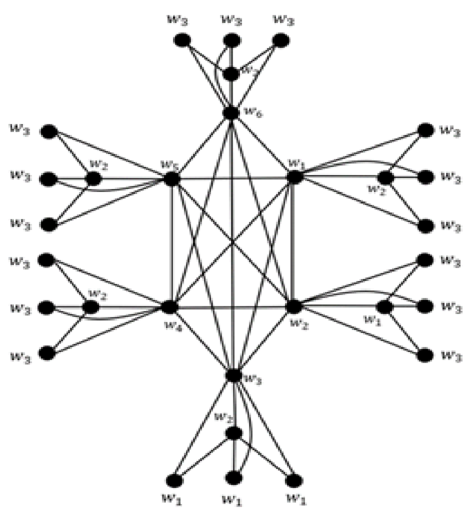

Figure 2. The vertex coloring for $K_6 \odot K_{1,3}$.

4.2 The Chromatic Number of Graph $K_{1,m} \odot K_n$

Proposition 4.2. The chromatic number of graph $K_{1,m} \odot K_n$, $m = 1,2,\ldots,p$ is $\chi\,(K_{1,m} \odot K_n) = n+1,\ for\ \ n = 1,\ 2,\ \ldots,\ k.$

Proof. Let b is a center vertex of $K_{1,m}$*and* $C = \{c_1,\ c_2,\ \ldots,\ c_m\}$ is the pendant vertices of $K_{1,m}$. Steps to color vertices $K_{1,m} \odot K_n$ are as follows:

1. Coloring the vertices in $K_{1,m}$ is done first. According to **Corollary 2.3**, $\chi\,(K_{1,m}) = 2$. Let the colors are $W = w_1, w_2$, where w_1 is the color for b and w_2 is the color for vertices in C.

2. Next, give color to the vertices in K_n at $K_{1,m} \odot K_n$. Since every vertex in K_n adjacent to a vertex in $K_{1,m}$, then only one color in W that can be used to color K_n. So, it needs $n-1$ new color such that K_n at $K_{1,m} \odot K_n$ can be colored minimally, that is by:

(a) Vertices in K_n which adjacent to the vertex b in $K_{1,m}$ are colored first. Since every vertex in K_n adjacent to the vertex b in $K_{1,m}$, then the color w_1 can not be used to color vertices in K_n. Then, the color w_2 is used to color one of the vertices in K_n. There are still $n-1$ vertices in K_n not colored yet. Those vertices are colored by $n-$ new color.

(b) Next, give color to the vertices in K_n which are connected to one of the vertices in C. Let a vertex in the C is c_1. Since every vertex in K_n adjacent to the c_1, then there is no vertex in K_n that can be colored with w_2. So, the color w_1 is used to color one of the vertices in K_n. There are still $n-1$ vertices in K_n not colored yet. Those vertices are colored by $n-1$ colors which have been used in K_n that adjacent to the vertex b.

(c) Giving color to the vertices in K_n that adjacent to vertices $c_1, c_2, \ldots, c_m$ is done by similar way to color K_n which adjacent to the vertex c_1 in $K_{1,m}$.

So, the smallest number of colors required to color the vertices of $K_{1,m} \odot K_n$ is $|W| + (n-1) = 2 + n - 1 = n + 1$ colors. The vertex coloring of graph $K_{1,m} \odot K_n$ can be defined by following function:

$$f(v) = \begin{cases} w_1, \quad v = \{b, a_{i1} | i = 1, 2, \ldots, m\} \\ w_2, \ v = \{c_i, a_{01} | i = 1, 2, \ldots, m\} \\ w_k, \ \ v = \{a_{ik-1} | i = 0, 1, 2, \ldots, m \ and \ k = 3, 4, \ldots, n+1 \end{cases}$$

where b is the center vertex in $K_{1,m}$, c_i are pendant vertices in $K_{1,m}$, and a_{ij} are vertices in K_n.

As an illustration, Figure 3 shows the vertex coloring of $K_{1,3} \odot K_5$, with $\chi(K_{1,3} \odot K_5) = 6$. The center vertex in $K_{1,3}$ is colored by w_1, the pendant vertices c_i, $i = 1, 2, 3$ are colored by w_2. Then, four copies of K_5 each colored by five colors, such that there are no two adjacent vertices in $K_{1,3} \odot K_5$ have the same color. □

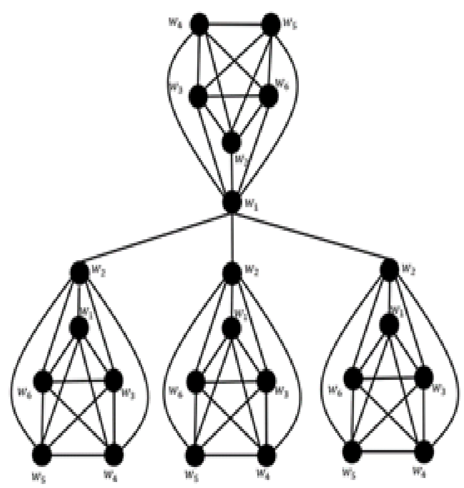

Figure 3. Vertex coloring of graph $K_{1,3} \odot K_5$.

4.3 The Vertex Coloring Visualization Program

Based on the function to color the vertices of $K_{1,m} \odot K_n$ and $K_n \odot K_{1,m}$, we constructed a program using Matlab software to show the coloring. Users can input the value of n and m. The output of the program is graph $K_{1,m} \odot K_n or K_n \odot K_{1,m}$, with its vertex coloring. Figure 4 shows the flowchart for the algorithm of the program. Figure 5 and figure 6, respectively, are output of the vertex coloring visualization program for graphs $K_n \odot K_{1,m}$ and $K_{1,m} \odot K_n$.

5. CONCLUSION

In this paper, we found that the chromatic number of the corona product of complete graph K_n and star graph $K_{1,m}$ is $\chi(K_n \odot K_{1,m}) = \begin{cases} 3, & n = 1, 2 \\ n, & n = 3, 4, \ldots, k \end{cases}$. In addition, the chromatic number of the corona product of star graph $K_{1,m}$ and complete graph K_n $is \chi(K_{1,m} \odot K_n) = n + 1,\ n = 1, 2, \ldots, k$. The chromatic numbers are verified by its visualization program.

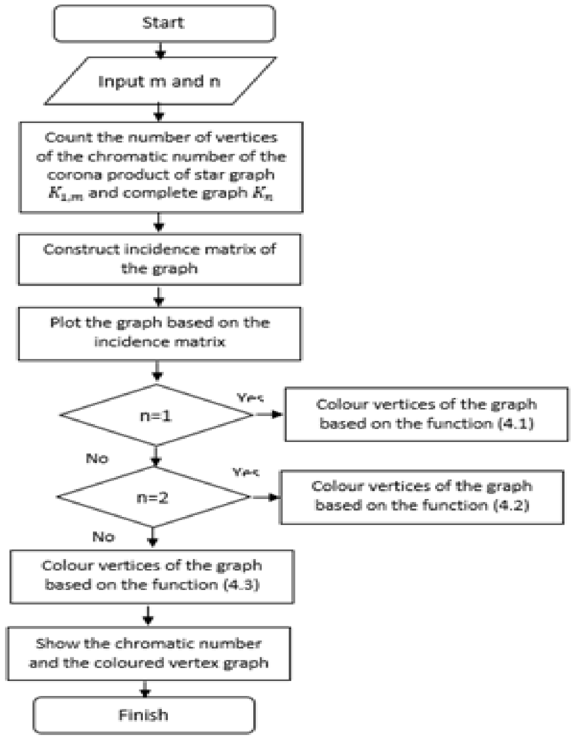

Figure 4. Flow chart of programme visualization of vertex coloring of graph $K_n \odot K_{1,m}$.

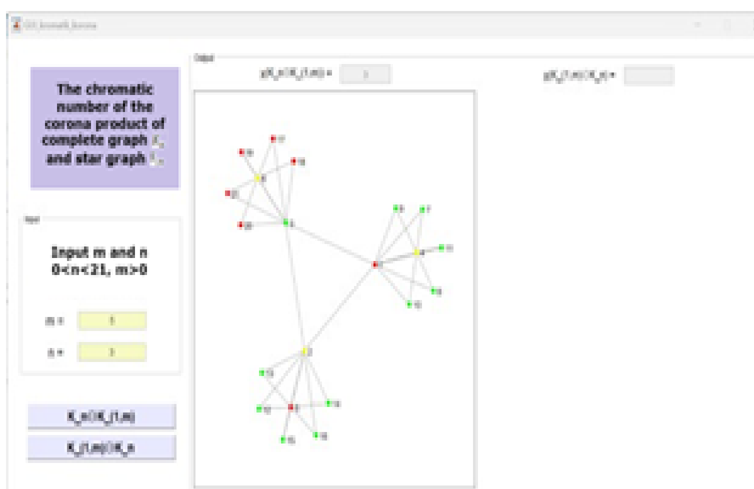

Figure 5. Output program for the vertex coloring $K_3 \odot K_{1,5}$.

For further research, the edge-chromatic number of corona product of complete graph K_n and star graph $K_{1,m}$ can be conducted.

REFERENCES

[1] J. A. Bondy and U. S. R. Murty, *Graph Theory*. Springer Publishing Company, Incorporated, 2008.

[2] A. S. Asratian, T. M. Denley, and R. Häggkvist, *Bipartite Graphs and Their Applications*. Cambridge University Press, 1998, vol. 131.

[3] F. Harary and R. Frucht, "On the corona of two graph," *Aequationes Mathematicae*, pp. 322–325, 1970.

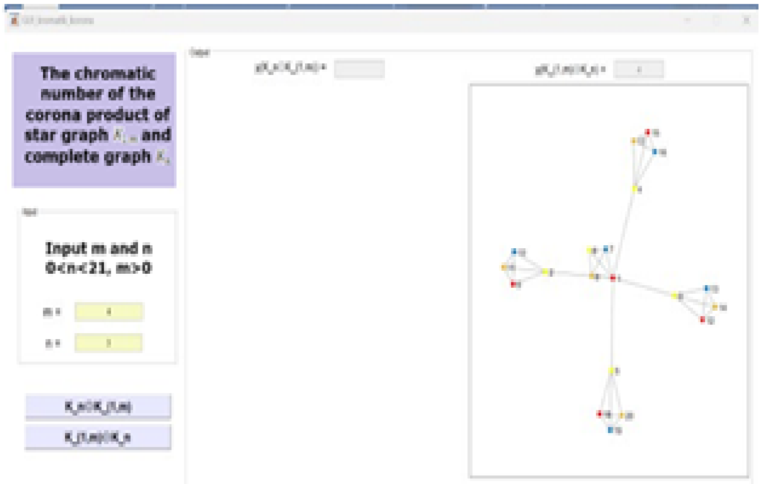

Figure 6. Output program for the vertex coloring $K_{1,4} \odot K_3$.

[4] E. G. Goodaire and M. M. Parmenter, *Discrete Mathematics with Graph Theory*, 2nd ed. Prentice-Hall, 2002.

[5] S. S. B. Ginting and M. Mulyono, "Bilangan kromatik dari graf hasil operasi korona pada graf bintang dan graf lingkaran," *Jurnal Riset Rumpun Matematika dan Ilmu Pengetahuan Alam*, vol. 2, no. 2, pp. 263–269, 2023.

[6] S. Mohan, J. Geetha, and K. Somasundaram, "Total coloring of the corona product of two graphs," *Australasian Journal of Combinatorics*, vol. 68, no. 1, pp. 15–22, 2017.

[7] P. Galinier, J. P. Hamiez, J. K. Hao, and D. Porumbel, "Recent advances in graph vertex coloring," in *Handbook of Optimization: From Classical to Modern Approach*, 2013, pp. 505–528.

[8] S. Simanjuntak and M. Mulyono, "Bilangan kromatik hasil operasi korona graf lingkaran dan graf kubik," *Karismatika*, vol. 7, no. 2, pp. 25–31, 2021.

[9] F. Buckley and M. Lewinter, *A Friendly Introduction to Graph Theory*. New Jersey: Pearson Education, 2003.