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Mean Labelling On Some Graph

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ABSTRACT
This paper aims to prove that some certain graphs, such as triangular book graphs, ladder graphs, snail graphs, and Cartesian product graphs between cycles and paths, belong to the category of mean graphs. We chose these graph classes because it is still an open problem. To prove the above, we use two main approaches, such as the axiomatic descriptive method and the pattern detection method. The axiomatic descriptive method is used to describe the basic properties of graphs and organise them into formal arguments, while the pattern detection method is used to observe and generalise the structural properties of graphs in a more exploratory manner. Based in the results of the analysis and proof, we conclude that the four classes of graphs studied, to be triangular book graphs, ladder graphs, snail graphs, and Cartesian product graphs of cycles and paths, are proven to satisfy the characteristics of mean graphs.

KEYWORDS
Graph labelling, mean labelling, mean.

1. INTRODUCTION

Graph theory is a mathematical discipline with diverse applications across every aspect of life. In this theory, objects are represented as vertices, while the relations between objects are depicted through edges [1]. The concept of graphs was first introduced by Leonhard Euler, a Swiss mathematician, in 1736 [2]. The solution to the Königsberg Bridge puzzle marked the beginning of graph theory. Leonhard Euler argued that it was impossible for someone to cross each bridge exactly once and return to their starting position. A graph $G(V,E)$ is formally described as a set of pairs $(V(G),E(G))$, where $V = V(G)$ is a non-empty finite set of vertices, and $E = E(G)$ is a set of edges that connect these vertices [3]. To the present day, studies on graph theory continue to evolve, including those related to the topic of graph labelling.

Graph labeling is the assignment of integers to graph components, such as vertices and edges, according to specific criteria [4]. The idea of graph labeling was initially presented in the mid-1960s [5]. Based on the domain of origin, graph labelling is classified into three types: vertex labelling (if the domain of origin of the graph labelling consists of graph vertices), edge labelling (if the domain of origin of the graph labelling consists of edges), and total labelling (if the domain of origin of the graph labelling consists of both vertices and edges) [6]. In recent years, over 350 graph labeling strategies have been researched in more than 3600 publications see [7]. Somasundaram and Ponraj in [8] introduced a graph labelling technique called mean labelling.

A graph that can be designated with mean labelling is referred to as a mean graph. Prior studies have demonstrated that certain graphs qualify as mean graphs, such as path, cycle, $K_{2,n}$, $K_2 + mK_1$, $K_n + 2K_2$, $C_m \cup P_n$, $P_m \times P_n$, $P_m \times C_n$, $C_m \odot K_1$, $P_n \odot K_1$, triangular snake, square snake, complete graph (K_n) with $n < 3$, $K_{1,n}$ with $n < 3$, double star graph $B_{m,n}$ with $m > n$, and friendship graph see [8], [9], [10], [11], [12], [13]. The complete mean labelling survey can be seen at [7].

Based on previous research, this paper discusses mean labelling on several graphs, such as triangular book graphs, ladder graphs, snail graphs, and $C_3 \times P_n$ graphs. We chose these graphs because it is still an open problems. This study aims to obtain mean labelling patterns in triangular book graphs, ladder graphs, snail graphs, and $C_3 \times P_n$ graphs. We hope that the results of this study can serve as additional references, especially on the topic of graph labelling.

2. LITERATURE REVIEW

Somasundaram and Ponraj [8] introduce the idea of mean labelling on graphs. Let G be a simple graph with p vertices and q edges, termed a mean graph if there exists an injective function f from the vertices of G to the set $\{0,1,\ldots,q\}$ such that when each edge uv is labelled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even, and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then the producing edge labels are all unique. Gayathri and Gopi [14] prove that if G is a mean graph, thus any vertex x with label $f(x) \in \{0,1,2,\ldots,n\}$ has number of adjacent vertex labels as follows.

1. $\frac{n+1}{2}$ when n is odd, $\frac{n}{2}$ when n is even if $f(x) = 0$.
2. $\frac{n+3}{2}$ when n is odd, $\frac{n+2}{2}$ when n is even if $f(x) \equiv 0 \; mod \; 4$ and $f(x) \neq 0$.
3. $\frac{n+1}{2}$ when n is odd, $\frac{n+2}{2}$ when n is even if $f(x) \equiv 1 \; mod \; 4$.
4. $\frac{n+3}{2}$ when n is odd, $\frac{n+2}{2}$ when n is even if $f(x) \equiv 2 \; mod \; 4$.
5. $\frac{n+1}{2}$ when n is odd, $\frac{n+2}{2}$ when n is even if $f(x) \equiv 3 \; mod \; 4$.

Previous researchers have identified some graphs that are classified as mean graphs. Vasuki and Nagarajan [15] show that the splitting graphs of paths and even cycles, $C_m \odot P_n$, $C_m \odot 2P_n$, and $C_n \cup C_n$ are mean graphs. Vaidya and Bijukumar [16] shown that mean graphs are those that are produced by mutually duplicating a pair of vertices from each cycle copy and a pair of edges from each cycle copy. Ramya and Jayenthi [17] labelled the mean on several tree graphs and obtained the result that graph T_n is a mean graph. Bailey and Barrientos [18] proved that the coronas $G \odot K_1$ and $G \odot K_2$ represent mean graphs, given that G is a mean graph. Gayathri and Gopi [19] proved that the subsequent graphs represent mean graphs: double triangular snake, double quadrangular snake, generalized antiprism, graphs created by connecting two vertices $K_{2,n}$ of degree n with an edge, and the graph resulting from C_n with vertices $v_1, v_2, \ldots, v_n$ by including an edge connecting v_i and v_{n-i+2} with $2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$. Kaneria and Meghpara [20] labelled the mean on some cycle graphs (C_n), bistar graphs (B_n), and double fan graphs (Df_n) and concluded that these graphs are mean graphs. The alternating triangular snake graph $A(Tn)$ and the alternating quadrilateral snake graph $A(Qn)$ have also been proven to be mean graphs [21].

Based on previous research, triangular book graphs, ladder graphs, snail graphs, and Cartesian cycle and path product graphs have not been identified as satisfying mean labelling or not. Therefore, in this paper, we will show that these graphs are mean graphs.

3. METHODOLOGY

This research is classified as exploratory research that aims to provide new insights and stimulate fresh ideas that can be used as additional references. To prove that triangular book graphs, ladder graphs, snail graphs, and $C_3 \times P_n$ graphs are mean graphs, we use an axiomatic descriptive approach and pattern identification methods. The steps in conducting this research are explained in detail as follows.

1. The first step in this research is to use the axiomatic descriptive method to explore several definitions, lemmas, or theorems related to mean labelling.
2. The second step is to select the graph class to be studied.
3. The third step is to perform labelling in accordance with the mean labelling conditions on the selected graph class at a certain order.
4. Use the pattern detection method to find the general function of mean labelling on the selected graph.
5. The labelling function obtained is then formed into a theorem and proven.

4. RESULT & DISCUSSION

This section presents the evidence about triangular book graphs, ladder graphs, snail graphs, and $C_3 \times P_n$.

4.1 Triangular book graphs

The triangular book graph $K_{1,1,n}$ with order n and size $2n+1$ is a complete tripartite graph $K_{r,s,t}$ with r and s vertices equal to one and n vertices on t. There are n copies of the cycle graph $C_3 (n \geq 2)$ in the triangular book graph $K_{1,1,n}$, where each cycle C_3 shares one edge, or in other words, each cycle C_3 has two identical vertices [22].

Theorem 4.1. For every $n \geq 2$, the triangular book graph $K_{1,1,n}$ is a mean graph.

Proof. The triangular book graph $K_{1,1,n}$ has the following notation for vertices and edges.

$$V(K_{1,1,n}) = \{v_0\} \cup \{v_0'\} \cup \{v_i; i = 1,\ 2, \ldots, n\}$$

$$E(K_{1,1,n}) = \{v_0 v_0'\} \cup \{v_0 v_i; i = 1,\ 2, \ldots, n\} \cup \{v_0' v_i; i = 1,\ 2, \ldots, n\}$$

Figure 1 illustrates how the vertices and edges of the triangular book graph $K_{1,1,n}$ are notated. We show that the graph of the

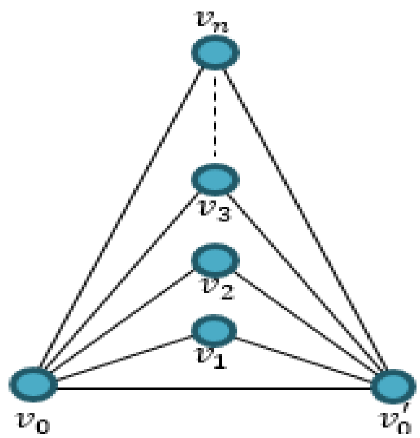

Figure 1. Triangular book graph

triangular book $K_{1,1,n}$ is a mean graph by defining a labelling function that satisfies the mean labelling condition. Define an injective function $f : V(K_{1,1,n}) \to \{0, 1, 2, \ldots, 2n+1\}$ as follows.

$$f(v_0) = 0$$
$$f(v_0') = 2n+1$$
$$f(v_i) = 2i; i = 1,\ 2, \ldots, n.$$

If f^* is the edge labelling constructed by f, then

$$f^*(v_0 v_0') = n+1$$
$$f^*(v_0 v_i) = i; i = 1,\ 2, \ldots, n$$
$$f^*(v_0' v_i) = n+i+1; i = 1,\ 2, \ldots, n$$

It is evident that each induced edge of the vertex labels has a bijective label, meaning that the mean labeling requirement is satisfied by the labeling functions f and f^*. It is proven that the triangular book graph $K_{1,1,n}$ is a mean graph □

As an example, **Figure 2** shows the mean labelling of the triangular book graph $K_{1,1,3}$. Based on **Figure 2**, it shows that the edge labels are bijective while the vertex labels remain injective, with the largest vertex label being 7, which is equivalent to the size of the triangle graph $K_{1,1,3}$. It is clear that this satisfies the mean labelling condition.

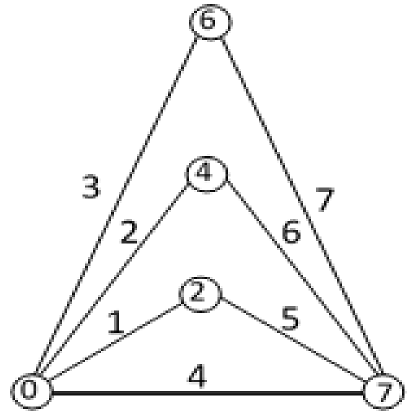

Figure 2. Mean labelling on graph $K_{1,1,3}$

4.2 Ladder graph

The ladder graph symbolised by L_n represents a simple connected graph constructed by the Cartesian product of the path graph P_n and the path graph P_2, or it can be written as $L_n = P_n \times P_2$ [3]. The ladder graph has order $2n$ and size $3n-2$.

Theorem 4.2. The ladder graph L_n represents a mean graph for each $n \geq 3$.

Proof. The vertices and edges of the ladder graph L_n are denoted as follows.

$$V(L_n) = \{u_k; k = 1,\ 2, \ldots, n\} \cup \{v_k; k = 1,\ 2, \ldots, n\}$$
$$E(L_n) = \{u_k u_{k+1}, v_k v_{k+1}; k = 1,\ 2, \ldots, n-1\} \cup \{u_k v_k; k = 1,\ 2,\ \ldots, n\}$$

An illustration of vertices and edges notation on a ladder graph (L_n) can be seen in **Figure 3**. In the same way, to show that the

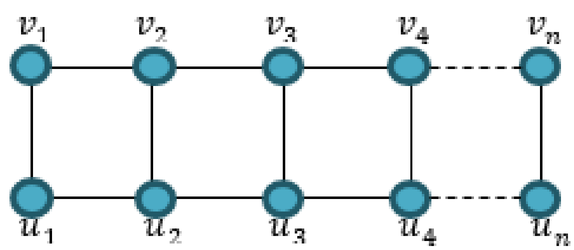

Figure 3. Ladder graph

ladder graph L_n represents a mean graph, we define a labelling function that satisfies the mean labelling condition on that graph. Determine an injective function $f : V(L_n) \to \{0,\ 1,\ 2, \ldots, 3n-2\}$ as follows.

$$f(u_k) = \begin{cases} 5k-5; k = 1, 2, \\ 3k-2; k = 3, 4, \ldots, n \end{cases}$$
$$f(v_k) = \begin{cases} 1; k = 1, \\ 3k-3; k = 2, 3, \ldots, n. \end{cases}$$

If f^* is the edge labelling constructed by f, then

$$f^*(u_k u_{k+1}) = 3k; k = 1,\ 2, \ldots, n-1.$$
$$f^*(v_k v_{k+1}) = 3k-1; k = 1,\ 2, \ldots, n-1.$$
$$f^*(u_k v_k) = 3k-2; k = 1,\ 2, \ldots, n.$$

It is evident that each edge's label, which results from the vertices' labeling, is bijective, meaning that the labelling functions f and f^* meet the mean labeling requirement. The ladder graph L_n with $n \geq 3$ has been proven as a mean graph. $\square$

To clarify **Theorem 4.2**, an example of mean labelling on the ladder graph L_4 is presented in **Figure 4**. Based on **Figure 4**,

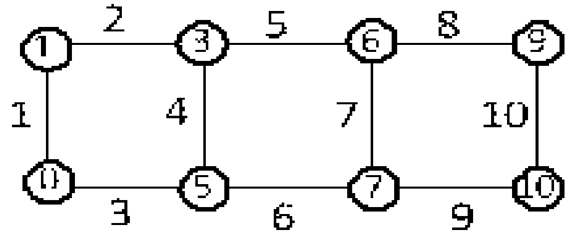

Figure 4. Mean labelling on ladder graph L_4

it shows that the edge labels are bijective while the vertex labels are injective, with the largest vertex label being 10, which is equal to the size of the ladder graph L_4. It is clear that this satisfies the mean labelling condition.

4.3 Snail graph

A snail graph is a graph that is created by identifying edges between the triangular graph $K_{1,1,n}$ with the cycle graph C_4, then one of the vertices of degree two in the cycle graph C_4 is identified with the central vertex of the star graph S_2. SI_n indicate a snail graph with $n+6$ vertices and $2n+6$ edges [23]. The identification of vertices from graphs G and H at vertices $x \in V(G)$ and $y \in V(H)$ is denoted by $(G \odot_{xy} H)$, resulting in a new graph created by connecting vertices x and y such that the new graph has $(|V(G)| + |V(H)| - 1)$ vertices and $(|E(G)| + |E(H)|)$ [24].

Theorem 4.3. The snail graph SI_n represents a mean graph for every $n \geq 1$.

Proof. Vertices and edges in the snail graph SI_n are represented by the following notation.

$$V\left(SI_n\right) = \{v_i; i \in [1, n+6]\}$$
$$E\left(SI_n\right) = \{v_i v_{n+3}; i \in [1, n]\} \cup \{v_i v_{n+4}; i \in [1, n]\} \cup v\left\{{}_{n+2}v_{n+4}, v_{n+3}v_{n+4}, v_{n+2}v_{n+5}, v_{n+3}v_{n+5}, v_{n+1}v_{n+2}, v_{n+6}v_{n+2}\right\}$$

Figure 5 illustrates the vertices and edges of the snail graph (SI_n). Similarly, to show that the snail graph (SI_n) qualifies as a

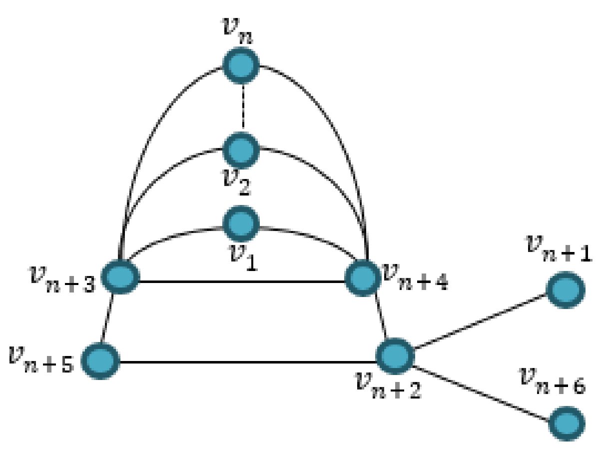

Figure 5. Snail graph

mean graph, we determine a labelling function that satisfies the mean labelling condition on that graph. Define an injective function $f: V(SI_n) \rightarrow \{0, 1, 2, \ldots, 2n+6\}$

$$f(v_{n+5}) = 1.$$
$$f(v_{n+3}) = 0.$$
$$f(v_{n+4}) = 2n+4.$$
$$f(v_{n+2}) = 2n+5.$$
$$f(v_{n+1}) = 2n+6.$$
$$f(v_{n+6}) = 2n+3.$$
$$f(v_i) = 2i+2; i = 1,\ 2, \ldots, n.$$

If f^* is the edge labelling constructed by f, then

$$f^*(v_{n+3}v_i) = i+1; i = 1,\ 2, \ldots, n.$$
$$f^*(v_{n+3}v_{n+4}) = n+2.$$
$$f^*(v_{n+4}v_i) = n+i+3; i = 1,\ 2, \ldots, n.$$
$$f^*(v_{n+4}v_{n+2}) = 2n+5.$$
$$f^*(v_{n+5}v_{n+2}) = n+3.$$
$$f^*(v_{n+2}v_{n+1}) = 2n+6.$$
$$f^*(v_{n+2}v_{n+6}) = 2n+4.$$

We can identify that each induced edge of the vertices' labelling is bijective, so that the labelling functions f and f^* satisfy the mean labelling condition. It is proven that the snail graph SI_n with $n \geq 1$ is a mean graph. □

As an example of **Theorem 4.3**, **Figure 6** shows the mean labelling of the snail graph SI_2. **Figure 6** shows that the vertex

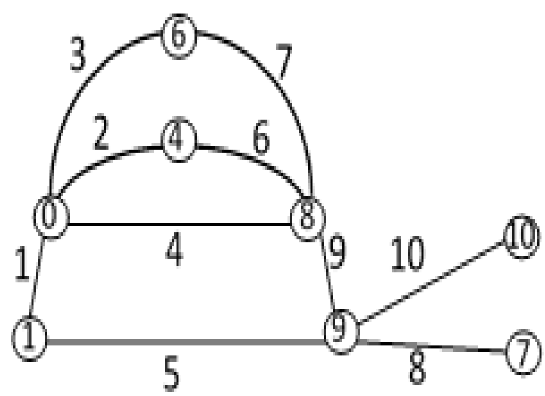

Figure 6. Mean labelling on snail graph SI_2

labels are injective while the edge labels are bijective, with the largest vertex label being 10, which is equal to the size of the SI_2 snail graph. It is clear that this satisfies the mean labelling condition.

4.4 Graph $C_3 \times P_n$

The graph $C_3 \times P_n$ has order $3n$ and size $6n-3$. The set of vertices of the graph $C_3 \times P_n$ is $(C_3 \times P_n) = \{v_i^k; i = 1,\ 2,\ 3; k = 1,\ 2,\ \ldots, n\}$ and the set of edges is $(C_3 \times P_n) = \{v_1^k v_2^k, v_2^k v_3^k, v_1^k v_3^k; k = 1,\ 2,\ \ldots, n\} \cup \{v_i^k v_i^{k+1}; i = 1,\ 2,\ 3; k = 1,\ 2,\ \ldots, n-1\}$. An example of vertex and edge notation in the graph $C_3 \times P_n$ can be seen in **Figure 7**.

Theorem 4.4. The graph $C_3 \times P_n$ represents a mean graph for every $n \geq 2$.

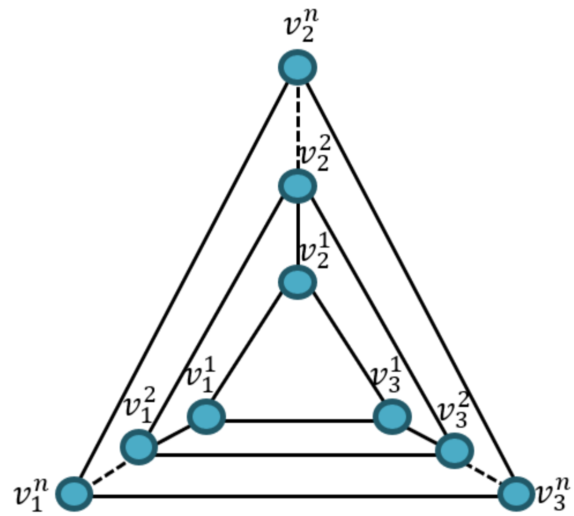

Figure 7. Graph $C_3 \times P_n$

Proof. By defining a labelling function that satisfies the mean labelling condition, we can show why the graph $C_3 \times P_n$ represents a mean graph.Define an injective function $f : V(C_3 \times P_n\ ) \to \{0,1,2,\dots,6n-3\}$. For $j = 1,2,...,n$

$$\left(v_1^k\right) = \begin{cases} 6k-6; k \equiv 1\ mod\ 3, \\ 6k-3; k \equiv 2\ mod\ 3, \\ 6k-4; k \equiv 0\ mod\ 3. \end{cases}$$

$$f\left(v_2^k\right) = \begin{cases} 6k-3; k \equiv 1\ mod\ 3, \\ 6k-4; k \equiv 2\ mod\ 3, \\ 6k-6; k \equiv 0\ mod\ 3. \end{cases}$$

$$f\left(v_3^k\right) = \begin{cases} 6k-4; k \equiv 1\ mod\ 3, \\ 6k-6; k \equiv 2\ mod\ 3, \\ 6k-3; k \equiv 0\ mod\ 3. \end{cases}$$

If f^* is the edge labelling constructed by f, then for $k = 1,\ 2,\dots,n$

$$f^*\left(v_1^k v_2^k\right) = \begin{cases} 6k-4; k \equiv 1\ mod\ 3, \\ 6k-3; k \equiv 2\ mod\ 3, \\ 6k-5; k \equiv 0\ mod\ 3. \end{cases}$$

$$f^*\left(v_1^k v_2^k\right) = \begin{cases} 6k-3; k \equiv 1\ mod\ 3, \\ 6k-5; k \equiv 2\ mod\ 3, \\ 6k-4; k \equiv 0\ mod\ 3. \end{cases}$$

$$f^*\left(v_1^k v_3^k\right) = \begin{cases} 6k-5; k \equiv 1\ mod\ 3, \\ 6k-4; k \equiv 2\ mod\ 3, \\ 6k-3; k \equiv 0\ mod\ 3. \end{cases}$$

For $k = 1,\ 2, \ldots, n$.

$$f^*\left(v_1^k v_1^{k+1}\right) = \begin{cases} 6k-1; k \equiv 1 \ mod\ 3, \\ 6k \quad\ ; k \equiv 2 \ mod\ 3, \\ 6k-2; k \equiv 0 \ mod\ 3. \end{cases}$$

$$f^*\left(v_2^k v_2^{k+1}\right) = \begin{cases} 6k \quad\ ; k \equiv 1 \ mod\ 3, \\ 6k-2; k \equiv 2 \ mod\ 3, \\ 6k-1; k \equiv 0 \ mod\ 3. \end{cases}$$

$$f^*\left(v_3^k v_3^{k+1}\right) = \begin{cases} 6k-2; k \equiv 1 \ mod\ 3, \\ 6k-1; k \equiv 2 \ mod\ 3, \\ 6k \quad\ ; k \equiv 0 \ mod\ 3. \end{cases}$$

We can identify that the label of each edge induced from the vertex label is bijective, so that the labelling functions f and f^* satisfy the mean labelling condition. It is proven that the graph $C_3 \times P_n$ with $n \geq 2$ represents a mean graph. □

To clarify Theorem 4, an example of mean labelling on the graph $C_3 \times P_2$ is presented in **Figure 8**. Based on **Figure 8**, we

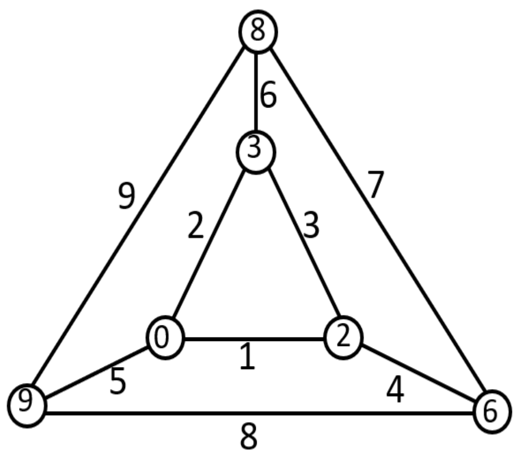

Figure 8. Mean labelling on the graph $C_3 \times P_2$

can see that the edge labels are bijective and the vertex labels are injective, with the largest vertex label being 9 or equal to the size of the graph $C_3 \times P_n$. It is clear that this satisfies the mean labelling condition.

5. CONCLUSION

According to the findings and discussion, it was found that the triangular book graph $K_{1,1,n}$ with $n \geq 2$, the ladder graph L_n with $n \geq 3$, the snail graph SI_n with $n \geq 1$, and the graph $C_3 \times P_n$ with $n \geq 2$ are mean graphs. In this study, we have not been able to find a labelling pattern in the graph $C_m \times P_n$ with $m \geq 4$, so other researchers can analyse this or look for new graph classes that are mean graphs.

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