Application of Extreme Value Distribution on Temperatures Data in South Sulawesi Province

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ABSTRACT, Climate change in recent years has resulted in extreme changes in temperature in Indonesia, especially in South Sulawesi Province. Extreme temperature changes will affect human and investment activities, energy use, and disaster events. Therefore, this study aims to model the maximum and minimum temperature data in South Sulawesi Province for the last 75 years. The data used came from observations of maximum and minimum temperatures from January 1945 to December 2020 and were analyzed using the Generalized Extrem Value distribution with the maxima block approach and the Generalized Pareto Distribution with the Peak Overtreshold (POT) approach. The results of the analysis show that these two models can be used to model extreme minimum and maximum temperature data in South Sulawesi Province with the right and optimal selection of blocks and thresholds. The results of the calculation of the return level every 5 years in the projection of the next 50 years show an increase in maximum and minimum temperatures which suggests the need to mitigate the risk of temperature change in order to adapt to climate change.

***Keywords**: Extrem distribution, POT, Block maxima, GEV, GPD, Return level, Temperature risk*

1. INTRODUCTION

Indonesia is recognized as one of the nations globally exhibiting an exceptionally high degree of susceptibility to climate change. As indicated in a report issued by the Asean Development Bank and the World Bank [1], Indonesia was positioned 97th among 181 nations in the 2020 ND-GAIN Country Index, which evaluates a nation's vulnerability to climate change and other global challenges, along with its preparedness to enhance resilience.

This country displays considerable susceptibility to the impact of climate change, compounded by geographical, social, and economic elements. This vulnerability is evident from several perspectives, such as the threat of natural disasters, effects on vital sectors, and social consequences. Furthermore, Indonesia ranks within the top third of nations facing high climate risk, with significant exposure to various forms of flooding and extreme heat. The severity of these hazards is projected to escalate with climate change, and in the absence of effective adaptation, the population's exposure will likely rise as well [1].

Climate change is significantly altering temperature patterns, especially in the ASEAN region including Indonesia. Temperatures in Indonesia are expected to continue to rise with an increase of between 0.8 to 1.4 ºC until 2050 [1]. Since October 2019, several BMKG stations in South Sulawesi have reported high temperatures such as Hasanuddin Meteorological Station in Makassar, recording a temperature of 38.8 degrees Celsius, followed by Maros Climatological Station with 38.3 degrees Celsius and Sangia Ni Bandera Meteorological Station with 37.8 degrees Celsius which shows the effect of hot temperatures on this area although it has not yet been categorized as hot waves [2]. The Climate Central has even reported that Makassar, the capital of South Sulawesi Province, is one of the top 5 cities in Southeast Asia with high heat for more than 90 days in 2024 [3].

This article will provide an overview of extreme models of temperature data for South Sulawesi Province that have rarely been studied. In reality, extreme temperatures have profound impacts on various aspects of society, ranging from economic and health outcomes to environmental and social dynamics. These impacts are increasingly significant as climate change intensifies the frequency and severity of extreme temperature events. Extreme temperature effects are diverse, affecting stock market performance[4], human mobility[5], mortality rate[6], the heightened reliance on fossil fuel resources and the consequent elevation in carbon dioxide emissions[7], also the occurrence of natural disasters [8]. Therefore, it is necessary to develop a predictive model to anticipate and reduce extreme events due to the increase in air temperature, as stated in this paper.

This research will use Extreme Value Theory to describe the extreme model of rainfall data in Indonesia. Some previous studies related to extreme values in Indonesian climate data include the application of the Gumbel distribution to analyze changes in extreme rainfall in Indonesia[9]; an altered symmetric logistic extreme value distribution for analyzing data on Indonesia's complex climate system [10] and analysis of extreme distribution in climate data in Indonesia by utilizing extreme index subset [11].

Climate change is likely to exacerbate the frequency and severity of extreme temperature phenomena, necessitating the implementation of extensive strategies for risk management and adaptation to these occurrences. This encompasses the enhancement of infrastructure resilience, the refinement of public health responses, and the formulation of policies aimed at addressing socioeconomic inequalities. A comprehensive understanding of temperature extremes is essential for enhancing climate risk assessments, informing disaster mitigation strategies, and facilitating sustainable development in communities that are particularly vulnerable.

2. LITERATURE REVIEW

Extrem Value Distribution

The extreme value distribution [12] was first introduced by Fisher and Tippet [13] nd has been widely used in modeling financial-related data such as stocks [14], insurance claims[15], extreme temperatures [16], climate data[17], wind speed[18], rainfall data[19], and axle load data[20].

Extreme value distributions in probability theory are described as limit distributions of the maximum and minimum values of as many as n i.i.d random variables with r normalized constant. There are 3 types of extreme value distributions when $n \to \infty$: Gumbel, Frechet, and Weibull distributions [12].

The Gumbel distribution is often used on data that represents maximum values. This is because the limits of the maximum value of mutually independent samples converge to a Gumbel distribution. The probability density function of this distribution is[21]:

$$f(x) = \frac{1}{\delta}\exp\left[\frac{\lambda - x}{\delta} - \exp\left(\frac{\lambda - x}{\delta}\right)\right], -\infty < x < \infty \tag{2.1}$$

The Fréchet distribution serves a similar purpose. The probability density function associated with this distribution is

$$f(x) = \frac{\beta\delta}{(x-\lambda)}\exp\left[-\left(\frac{\delta}{x-\lambda}\right)^{\beta}\right], \left(\frac{\delta}{x-\lambda}\right)^{\beta-1}, \quad x > \lambda \tag{2.2}$$

In contrast, the Weibull distribution is frequently encountered in practical scenarios that involve observational data characterized by minimum values. Within a population exhibiting a finite left tail, the limit of the minimum independent sample is expected to converge towards a Weibull distribution, which is represented by the following probability density function

$$f(x) = \frac{\beta}{\delta}\exp\left[-\left(\frac{x-\lambda}{\delta}\right)^{\beta}\right]\left(\frac{x-\lambda}{\delta}\right)^{\beta-1}, \quad x > \lambda \tag{2.3}$$

General Extreme Value (GEV) Distribution

The Generalized Extreme Value (GEV) distribution constitutes a limiting distribution for random variables derived from maximal samples that exhibit mutual independence and identical distribution. This GEV unifies three antecedent distributions, specifically Gumbel, Fréchet, and Weibull. The distribution function of the Generalized Extreme Value (GEV) is

$$G(x \mid \theta) = \begin{cases} \exp\left[-\left[1+z\right]\left(\frac{x-\mu}{\sigma}\right)^{-\frac{1}{z}}\right], & z \neq 0 \\ \exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right], & z = 0 \end{cases} \tag{2.4}$$

characterized by its location μ and scale σ parameters as shown in equation (2.4) with

$\theta=(\mu,\sigma,z)$ and density probability function [22]

$$f(x\mid\theta)=\frac{1}{\sigma}t^{z+1}\left(x\mid\theta\right)\exp\left[-t\left(x\mid\theta\right)\right] \quad (2.5)$$

where

$$t\left(x\middle|\Theta\right)=\begin{cases}\left[1+z\left(\frac{x-\mu}{\sigma}\right)\right]_{+}^{-\frac{1}{z}}, z\neq 0\\ \exp\left(\frac{z-\mu}{\sigma}\right), z=0\end{cases}$$

Generalized Pareto (GP) Distribution

An occurrence may be classified as an extreme event if its magnitude surpasses a specified threshold[22]. The Generalized Pareto (GP) distribution is employed when the values of a random variable exceed or fall short of a specified threshold. The GP distribution encompasses two distinct forms, characterized by the parameters λ and κ, which represent the scale and shape, respectively[21]:

1. Maximal GPD with distribution function and probability density function as follows:

$$G(x;\lambda;\kappa)=\begin{cases}1-\left(1-\frac{\kappa x}{\lambda}\right)^{\frac{1}{\kappa}};\left(1-\frac{\kappa x}{\lambda}\right)\geq 0, & \kappa\neq 0,\lambda>0\\ 1-\exp\left(-\frac{x}{\lambda}\right);x\geq 0,\kappa=0,\lambda>0\end{cases} \quad (2.7)$$

and

$$f(x;\lambda,\kappa)=\begin{cases}\frac{1}{\lambda}\left(1-\frac{\kappa x}{\lambda}\right)^{\left(\frac{1}{\kappa}\right)-1};\left(1-\frac{\kappa x}{\lambda}\right)\geq 0, & \kappa\neq 0,\lambda>0\\ \frac{1}{\lambda}\exp\left(-\frac{x}{\lambda}\right);x\geq 0,\kappa=0,\lambda>0\end{cases} \quad (2.7)$$

2. GPD minima with distribution function and probability density function as follows:

$$G(x;\lambda;\kappa)=\begin{cases}\left(1+\frac{\kappa x}{\lambda}\right)^{\frac{1}{\kappa}};\left(1+\frac{\kappa x}{\lambda}\right)\geq 0, & \kappa\neq 0,\lambda>0\\ \exp\left(\frac{x}{\lambda}\right);x\leq 0,\kappa=0,\lambda>0\end{cases} \quad (2.8)$$

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$$f(x;\lambda,\kappa)=\begin{cases}\frac{1}{\lambda}\left(1+\frac{\kappa x}{\lambda}\right)^{\left(\frac{1}{\kappa}\right)-1};\left(1+\frac{\kappa x}{\lambda}\right)\geq 0, & \kappa\neq 0,\lambda>0\\ \frac{1}{\lambda}\exp\left(\frac{x}{\lambda}\right);x\leq 0,\kappa=0,\lambda>0\end{cases} \quad (2.9)$$

Block Maxima and Peak Overtreshold

The block maxima (BM) methodology and the peak-over-threshold (POT) technique represent two essential methodologies within the domain of extreme value statistics. The BM methodology emphasizes the identification of the maximum value within a designated block of data, whereas the POT technique leverages observations that surpass a specified threshold, thereby enhancing the efficiency in the utilization of extreme values. The selection between these two methodologies may be influenced by the inherent characteristics of the data, such as the presence of independence or serial dependence, along with the specific statistical objectives, which may include the estimation of quantile rates or rates of return[23].

Return Level

In the GEV model, the estimate of the extreme quantile of the annual maximum distribution can be obtained using the equation

$$z_p = \begin{cases} \mu - \frac{\sigma}{z}\left[1 - \left\{-\log\left(1-p\right)\right\}^{-z}\right]; z \neq 0, \\ \mu - \sigma \log\left\{-\log\left(1-p\right)\right\}; z = 0 \end{cases} \quad (2.10)$$

with $G\left(z_p\right) = 1 - p$ and z_p is the return level associated with the return period $\frac{1}{p}$. The z_p level is expected to be exceeded on average $\frac{1}{p}$ once every year. More precisely, the annual maximum value in any specified year surpasses z_p with a certain probability of p. The same goes for $N-$years retun levels for the GPD models for n_y observations setiap tahunnya and exceedances of the threshold u by a variable X with $x > u$, is defined as follows[24]:

$$z_N = u + \frac{\sigma}{z}\left[\left(Nn_y \Pr\left\{X > u\right\}\right)^{z} - 1\right] \quad (2.11)$$

3. METHODOLOGY

The investigation employed air temperature datasets originating from South Sulawesi Province, encompassing monthly records from the year 1945 to 2020, which were sourced from the website www.keaggle.com. This dataset incorporates recorded maximum and minimum temperatures, expressed in degrees Celsius, on a monthly basis.

The procedures for implementing extreme value distribution to temperature datasets are as follows:

1. Identification of the descriptive statistics pertinent to the dataset.
2. Modeling the extrema of temperature data by employing Generalized Extreme Value (GEV) and Generalized Pareto Distribution (GPD).
3. Estimation of parameters for both GEV and GPD utilizing the Maximum Likelihood Estimation (MLE) technique.
4. Evaluating the appropriateness of the dataset with respect to the GEV or GPD model through the analysis of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values.
5. Ascertainment of the long-term return level associated with the temperature dataset.
6. Examination of the Mean Residual Life (MRL) plot in relation to the GPD model.
7. Assessment of the suitability of GEV and GPD extremal models based on diagnostic tests and MRL plots.

4. RESULT & DISCUSSION

Descriptive Statistics

The monthly maxima and minima of temperature data were subjected to a thorough analysis employing descriptive statistical methods to elucidate their distribution, variability, and measures of central tendency as demonstrated in **Table 4.1.** Monthly maximum temperatures in South Sulawesi province tend to be centered around 30.21–30.27°C, with most monthly temperatures falling between 29.86°C and 30.57°C. There exist certain months characterized by elevated temperatures, with values attaining 32.16°C.

In regard to the minimum temperature, it is predominantly situated within the range 19.57 – 19.39°C with the majority of the minimum temperatures recorded on a monthly basis falling between 18.87°C and 19.83°C. There were several instances throughout the observation period characterized by notably lower temperatures, culminating in a minimum 17.69°C. The graphical representations depicting the maximum and minimum temperatures for the period spanning 1945 - 2020 in South Sulawesi are illustrated in **Figures 4.1** and **4.2.**

GEV Models

In this study, the GEV model is applied to analyze monthly maximum and minimum temperature data to estimate the probability of future extreme events and identify long-term climate change patterns. GEV parameter estimation is performed using the Maximum Likelihood Estimation (MLE) method which is known to be robust in handling data with limited

Table 4.1 Air temperature data in South Sulawesi Province 1945-2020 (in degrees Celsius)

Data	**Mean**	**Mininimum**	**Maximum**	**1st Quartile**	**2nd quartile**	**3rd Quartile**
Minimum temperature	19.39	17.69	21.29	18.87	19.57	19.83
Maximum temperature	30.27	28.78	32.16	29.86	30.21	30.57

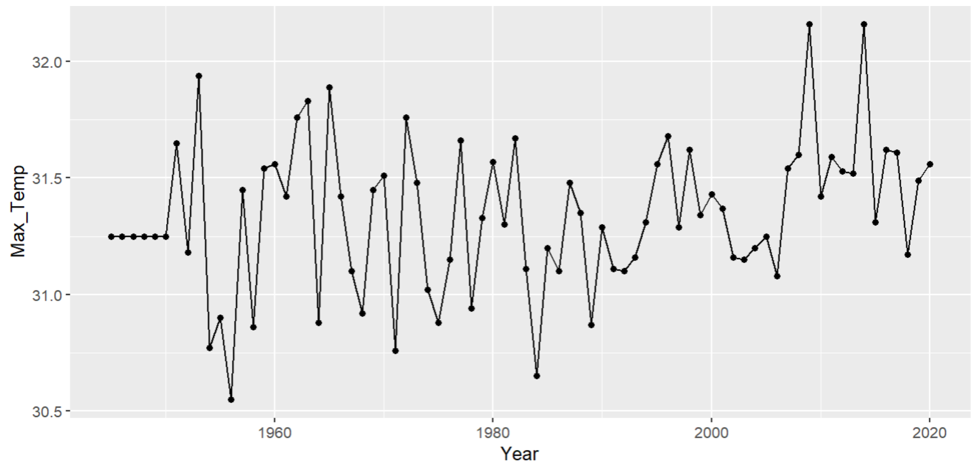

Figure 4.1 Maximum recorded air temperature in South Sulawesi Province from the year 1945 to 2020 (expressed in degrees Celsius)

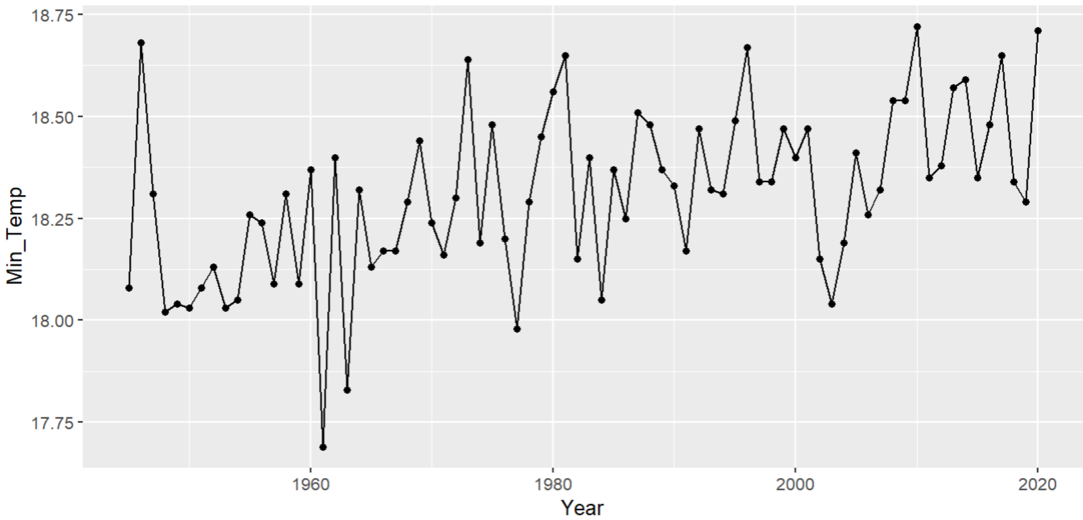

Figure 4.2 Maximum recorded air temperature in South Sulawesi Province from the year 1945 to 2020 (expressed in degrees Celsius)

Table 4.2 The estimation of the Generalized Extreme Value (GEV)'s parameters model within the context of South Sulawesi Province during the temporal span of 1945 to 2020 (in degrees Celsius).

Temperatures	**Parameters**	**Value**	**Error Standard**
Maximum	Location	31.2207869	0.03982328
	Scale	0.3185137	0.02724093
	Shape	-0.2493291	0.06106147
	AIC	49.07377	
	BIC	56.06597	
Minimum	Location	18.2472038	0.02740794
	Scale	0.2169276	0.01975483
	Shape	-0.3770490	0.07814547
	AIC	-18.84952	
	BIC	-11.85732	

sample size. The parameter estimation results are shown in **Table 4.2.**

Based on the findings from the parameter estimation of the Generalized Extreme Value (GEV) distribution for air temperature data in South Sulawesi Province from 1945 to 2020, distinct parameter values were derived for both maximum and minimum temperatures. For the annual maximum temperature, the location parameter (μ) of 31.22°C represents the central point of the extreme distribution, accompanied by a scale parameter (σ) of 0.318 that illustrates the characteristics of the extreme data distribution, and a shape parameter (z) of -0.249 which is negative. This negative shape value suggests that the distribution falls under the left Gumbel-type (Weibull-type) GEV classification, indicating the presence of an upper limit, signifying that there exists a maximum annual temperature value that will not be surpassed theoretically. The relatively low AIC and BIC values (49.07 dan 56.06) further reinforce the model's appropriateness for the data.

Meanwhile, regarding the annual minimum temperature, the location parameter is set at 18.25°C, accompanied by a scale of 0.217, and a shape parameter of -0.377. A more negative shape parameter suggests a shorter tail in the distribution, signifying that occurrences of extreme minimum temperatures are confined within a specific range. This lends support to the theory that extreme minimum temperatures in this area tend to exhibit a distinct lower threshold. The AIC (-18.85) and BIC (-11.86) values are even lower than those associated with the maximum temperature model, suggesting that the GEV model offers an exceptional fit for the minimum temperature dataset. The relatively small standard errors for all parameters indicate a high level of precision in the estimates.

The results of the analysis with extreme diagnostic plots of annual maximum temperature data using the Generalized Extreme Value (GEV) approach and the Block Maxima method are shown in **Figures 4.2** and **4.3**

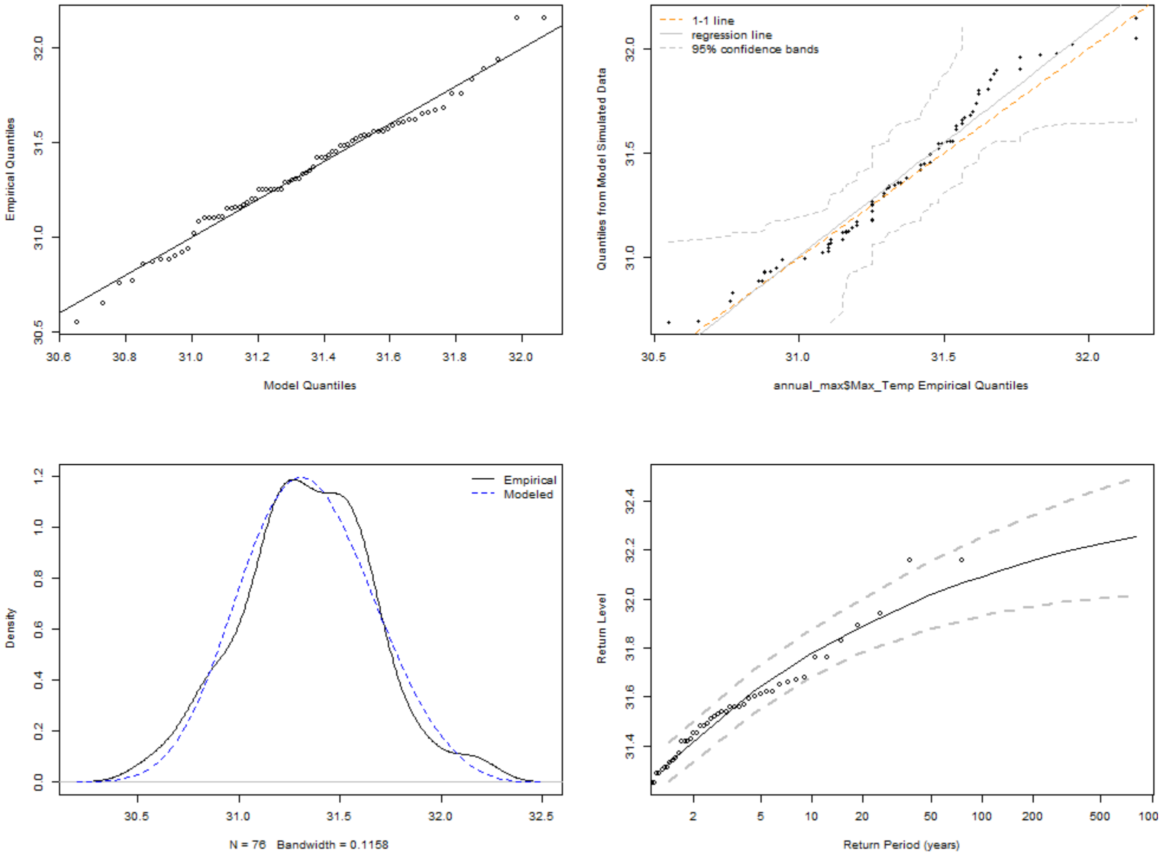

Figure 4.3 Diagnostic Plots for Maximum Temperature

The four graph panels from both graphs illustrate that the GEV model effectively captures the extreme behavior of maximum temperature. The QQ-plot (top left) and the probability plot (top right) reveal that the majority of empirical quantiles align with the quantiles projected by the model, as evidenced by the data points that closely follow the 1:1 line

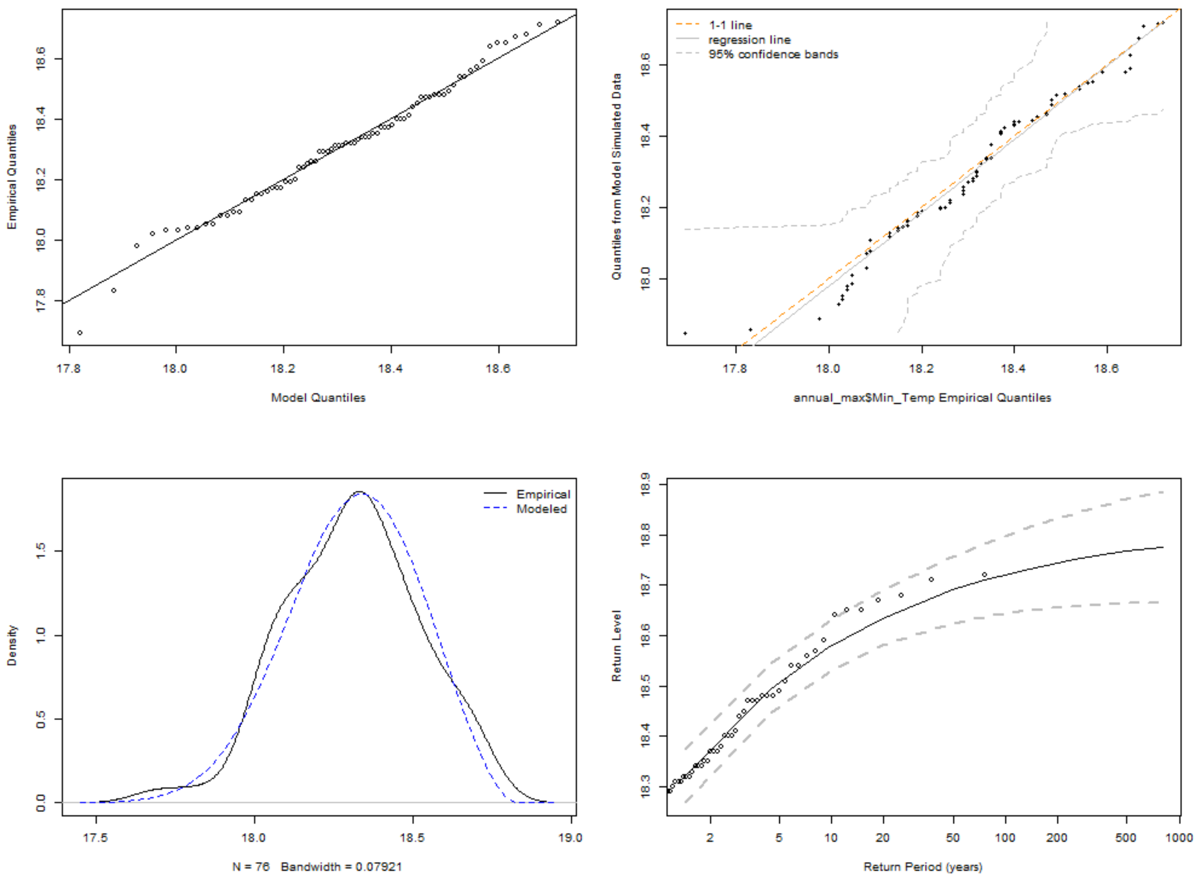

Figure 4.4 Diagnostic Plot untuk Temperature Minimum

and lie within the 95% confidence interval. This suggests that the assumption of the GEV distribution is adequately justified for this dataset. In addition, the density plot (bottom left) indicates that the model's distribution closely approximate the shape of the empirical distribution, with a strong alignment around the central value.

The return level chart (located in the bottom right corner) illustrates the logarithmic correlation between the return period and the extreme temperature levels, offering an estimation of the highest temperature that could potentially occur within a specific timeframe (for instance, 50 or 100 years). This chart holds significant value in the realm of extreme climate risk management, as it supplies insights that can aid in long-term strategies for anticipating possible future fluctuations in temperature, whether spikes or drops.

The return level values for monthly maximum and minimum temperatures can also be determined using equation (2.10) over a 50-year period, resulting in long-term temperature projections for South Sulawesi as shown in **Table 4.3.** The results show that for both maximum and minimum temperatures, the return level increases as the recurrence period increases. For example, the maximum temperature is estimated to reach 32.02°C in a 50-year recurrence period, while the extreme minimum temperature is estimated to reach 18.69°C in the same period. This increase in the return level reflects the increasing likelihood of more extreme temperature events over time.

Table 4.3 The return level of air temperatures in South Sulawesi Province from 1945 to 2020 (in degrees Celsius) using the GEV model

Period (in years)	**Return Level**	
	Maximum	**Minimum**
5	31.61937	18.49572
10	31.76935	18.57626
15	31.84237	18.61260
20	31.88910	18.63480
25	31.92282	18.65029
30	31.94886	18.66194
35	31.96990	18.67114
40	31.98743	18.67868
45	32.00239	18.68500
50	32.01538	18.69041

This pattern is important for estimating the risk of future extreme temperatures and their implications for various sectors, such as agriculture, health, and infrastructure. The fact that extreme temperatures are increasing, despite being gradual, highlights the need for climate adaptation policies that the long-term trends of extreme temperature changes. The GEV model used provides a robust statistical framework for predicting the likelihood of extreme events and can serve as a foundation for developing climate risk mitigation strategies in this region.

Model GPD

Subsequently, parameter estimation was conducted similarly utilizing the GP distribution, setting a maximum threshold at the 95th percentile and a minimum threshold at the 5th percentile. The results of the parameter estimation are presented in **Table 4.4**

Tabel 4.4 The estimation of the Generalized Pareto (GP)'s parameters model within the context of South Sulawesi Province during the temporal span of 1945 to 2020 (in degrees Celsius).

Temperatures	Parameter	Nilai	Standar Eror
Maximum	Scale	0.332629	0.0615687
	Shape	-0.299557	0.1203062
	AIC	-32.82731	
	BIC	-29.17003	
Minimum	Scale	0.1745138	0.03126016
	Shape	-0.1970075	0.10581862
	AIC	-82.73372	
	BIC	-79.07644	

The results of the Generalized Pareto (GPD) distribution estimation on extreme temperature data in South Sulawesi Province show that the shape parameter values for maximum temperature (-0.2996) and minimum temperature (-0.1970) are both negative. This indicates that the tail of the extreme distribution is bounded, meaning there is a theoretical maximum or minimum threshold for extreme temperatures that will not be exceeded. The scale parameter for maximum temperature (0.333) is larger than that for minimum temperature (0.175), indicating that the distribution of extreme values for maximum temperature is wider or more varied compared to minimum temperature.

The exceptionally low AIC and BIC values (notably for minimum temperature: AIC = -82.73) suggest that the GPD model fits the extreme temperature data remarkably well. The relatively low standard errors for both parameters further imply that the parameter estimates are stable and statistically significant. In summary, these findings demonstrate that the Peaks Over Threshold method using the GPD model is highly effective for examining the distribution of extreme temperature events, especially when focusing on values that surpass a specific threshold, such as in the analysis of short- and medium-term extreme climate risk.

The results of the analysis using extreme diagnostic plots on annual maximum temperature data using the Generalized Pareto Distribution (GPD) approach are shown in **Figures 4.4** and **4.5. T**he two plots in Figures 4.4 and 4.5 show diagnostic plots from Generalized Pareto (GPD) distribution modeling for extreme annual maximum and minimum temperature data using the Peaks Over Threshold (POT) approach.

QQ-plots and probability plots in both cases show that the empirical quantiles are fairly consistent with the model quantiles, with most points lying close to the 1:1 line and within the 95% confidence interval. This indicates that the GPD model can capture the distribution patterns of extreme data fairly well, both for high and low temperatures. However, there is a slight deviation in the upper tail, particularly in the maximum temperature data, suggesting that the model may be somewhat less accurate in modeling the highest extreme values.

The density plot shows that the model and empirical distributions have similar shapes, but there are differences in the peaks of the distributions, particularly at maximum temperatures, where the model produces a more symmetrical distribution than the actual data. Meanwhile, the return level plot shows that both maximum and minimum extreme temperatures experience an increase in return level as the return period increases, albeit with a flattening trend. This indicates a natural limit on the extreme temperatures that may occur in the future, consistent with the shape of the GPD, which has a negative shape parameter. Overall, the GPD model provides a reasonably reliable

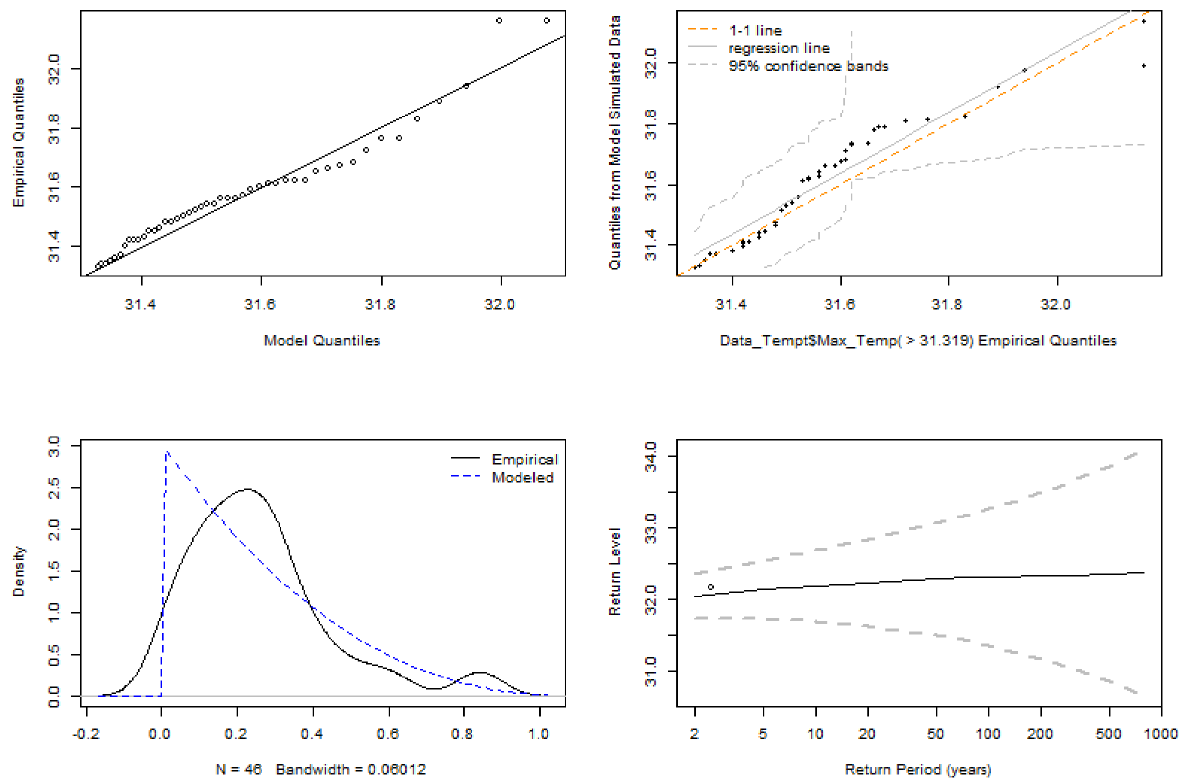

Figure 4.4 Diagnostic Representation for Maximum Temperature Utilizing Generalized Pareto Distribution

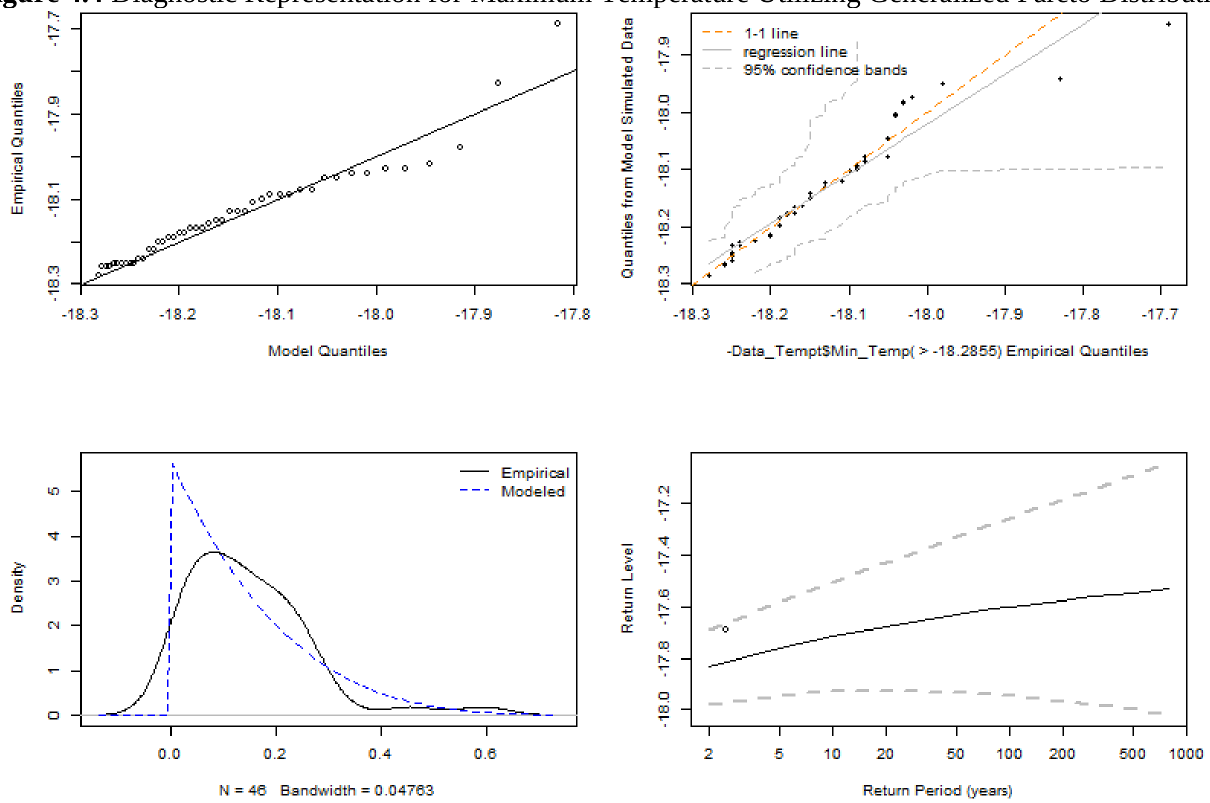

Figure 4.5 Diagnostic Representation for Minimum Temperature Utilizing Generalized Pareto Distribution

estimate of the distribution of extreme temperature events in the study area and can be used to support long-term climate risk analysis.

The return level values based on equation (2.11) for the GPD model are shown in the following Table 4.4

Table 4.5 The return level of air temperatures in South Sulawesi Province from 1945 to 2020 (in degrees Celsius) using the GPD model

Periode (in years)	Return Level	
	Maximum	Minimum
5	32.14295	17.76306
10	32.19666	17.71667
15	32.22328	17.69234
20	32.24030	17.67621
25	32.25253	17.66432
30	32.26193	17.65498
35	32.26948	17.64734
40	32.27575	17.64091
45	32.28108	17.63538
50	32.28569	17.63054

distribution. The results show that as the return period increases from 5 to 50 years, the maximum extreme temperature gradually increases from 32.14°C to 32.29°C, while the minimum extreme temperature actually decreases from 17.76°C to 17.63°C. This pattern reflects the characteristics of a GPD distribution with a negative shape parameter, which results in a bounded tail distribution, where increases in return levels tend to flatten out due to the presence of a theoretical upper bound.

The Best Model

Based on the AIC and BIC values in **Tables 4.2** and **4.4**, it can be seen that for both maximum and minimum temperatures, GPD has lower AIC and BIC values than GEV. Statistically, models with lower AIC and BIC values are considered better because they can explain the data efficiently without excessive complexity. In this case, the GPD model is more suitable for modeling extreme air temperature data in South Sulawesi Province, both for maximum and minimum values, compared to the GEV model.

However, it should be noted that the selection of thresholds in the GPD model and block periods in the GEV model will affect parameter estimation in both models. The maximum block (for GEV) is selected based on climatological logic (e.g., annually for extreme temperatures), and the threshold (for GPD) is determined using a graphical approach such as the mean residual life plot as shown in Figures **4.6** and **4.7**.

In the MRL Plot graph for Maximum Temperature, it is evident that the mean excess line steadily declines without exhibiting a notable flat section within the threshold range of 29–31.2°C. In the GPD methodology, it is crucial to identify the threshold range where the line appears to be relatively linear and maintains a stable horizontal position, signifying the point at which the GPD distribution assumption starts to hold true.

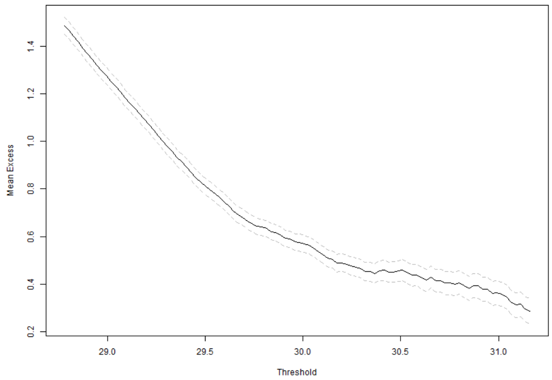

Figure 4.6 Mean Residual Life (MRL) Plot for Maximum Temperature Data

In the meantime, the trajectories on both graphs appear to consistently decline (curvilinear), lacking any definitive signs of stability. This suggests that there isn't a specific optimal threshold — thus, GPD modeling might not be the most suitable method for this maximum temperature data, or additional threshold sensitivity analysis could be required. Under these circumstances, a maximum block (annual) GEV model might present a more effective solution

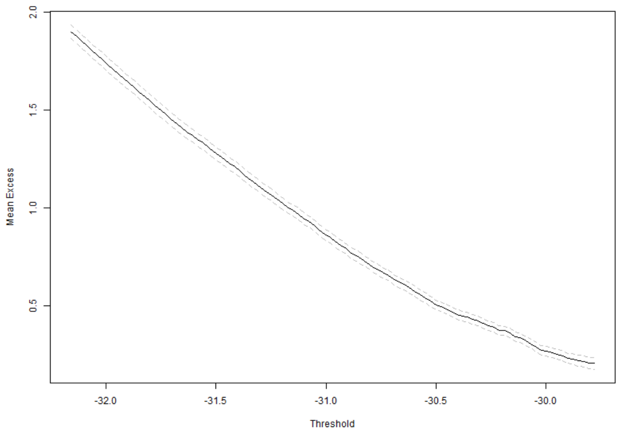

Figure 4.7 Mean Residual Life (MRL) Plot for Minimum Temperature Data

5. CONCLUSION

The analysis of the GEV and GPD models applied to maximum and minimum temperature data in South Sulawesi Province from January 1945 to December 2020 demonstrates the appropriateness of the extreme model for temperature variations in this region over the past 75 years.

The GEV distribution indicates a rising trend in maximum and minimum temperatures, with low AIC and BIC values observed for minimum temperatures. In contrast, the GPD distribution reveals lower AIC and BIC values than the GEV model, suggesting that it is a more precise model. Nevertheless, the MRL plot used to assess the GPD model's threshold illustrates a lack of values that exhibit a relatively linear or horizontally stable pattern, indicating that the selection of an optimal threshold will require careful consideration in the future. This approach can also be applied to the GEV model by determining the suitable block period.

The return levels of both models forecast an increase in maximum and minimum temperatures over the next 50 years. Hence, it is essential to plan for risk mitigation in response to these temperature changes to adapt to upcoming extreme climate shifts.

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