

Analysis of Catfish Production Growth Using Bernoulli's Differential Equation

Mey Gracia Sitanggang^{1*}, Tio Arini Pasaribu², Rawiyah³, Maria Adesuryani Purba⁴, Lahnida Sitinjak⁵, Hamidah Nasution⁶

^{1,2,3,4,5,6} *Department of Mathematics, State University of Medan, Indonesia*

*Corresponding author: meysitanggang565@gmail.com

*Submission date: 27 September 2025, Revision: 27 October 2025, Accepted: 08 November 2025

ABSTRACT

Catfish (*Clarias* sp.) is one of the most widely cultivated aquaculture commodities in Indonesia, playing an important role in food security and local economic development. This study applies Bernoulli's differential equation, which reduces to a logistic model, to evaluate catfish production in Indonesia from 2019 to 2025. Secondary data were obtained from the Ministry of Marine Affairs and Fisheries. Model parameters, including carrying capacity, intrinsic growth rate, and initial production, were estimated to fit the observed data. The results show that the logistic model effectively represents production trends between 2020 and 2024, with an average annual growth rate of approximately 4%. However, the model fails to capture the sharp decline observed in 2025, when actual production dropped by nearly 47%. This discrepancy indicates the presence of external non-linear factors, such as disease, environmental stress, or distribution disruptions, that were not included in the mathematical framework. Therefore, Bernoulli's differential equation provides a useful baseline for analyzing production growth under normal conditions, while highlighting the need to integrate ecological and managerial considerations for more accurate long-term predictions. These findings can inform management strategies related to disease prevention, environmental monitoring, and supply chain stabilization to mitigate future production declines.

KEYWORDS

Bernoulli differential equation, Logistic model, Catfish production, Mathematical modeling, Aquaculture.

1. INTRODUCTION

Fisheries play a crucial role in maintaining food security and supporting the community's economy, especially in Indonesia, which is rich in aquatic resources [1]. One of the fishery commodities with high economic value is catfish (*Clarias* sp.), which is widely cultivated by fish farmers from small to large scale [2]. In 2023, catfish production in Indonesia reached over 1.1 million tons, contributing significantly to the national aquaculture output [3]. The growth of catfish production is influenced by various factors, including the number of fingerlings, feed quality, environmental conditions, and cultivation management. Therefore, understanding the dynamics of catfish production growth is very important for increasing efficiency and planning sustainable production [4].

The Bernoulli differential equation is suitable for modeling production growth because it can incorporate non-linear factors such as resource limitations and environmental carrying capacity, which are critical in aquaculture. With this approach, changes in catfish production over time can be analyzed quantitatively, allowing for future production predictions based on existing data. This model can also be used to evaluate the influence of various external factors on the production growth rate, which helps in decision-making related to catfish cultivation management [5].

To address these limitations, this study applies the Bernoulli Differential Equation, which is suitable for modeling production

growth because it can incorporate non-linear factors such as resource limitations and environmental carrying capacity, which are critical in aquaculture¹. When its power value (n) is set to 2, the Bernoulli Equation reduces to the Logistic model, an ideal mathematical framework for analyzing production growth that approaches a carrying limit. Catfish production data from 2019 to 2025 is used to model the growth.

This analysis aims to achieve three specific objectives: estimate key model parameters, including the carrying capacity (K) and intrinsic growth rate (r) to fit the observed data, validate the model's performance against historical production trends (2020-2024), and analyze the significant anomaly observed in 2025, which indicates the strong influence of external, non-mathematical factors (such as disease or environmental stress) on production outcomes. The findings from this analysis are expected to provide a deeper understanding of production trends and establish a foundation for more effective management strategies in the future.

2. LITERATURE REVIEW

2.1 Differential Equations

A differential equation is a mathematical expression that relates a function to its rate of change (derivative) [5]. These equations are widely applied to model various phenomena in nature, such as in physics, biology, economics, and engineering. In the context of biology, particularly fisheries, differential equations are useful for modeling various aspects such as fish population growth, the spread of disease among fish, and the production of livestock or fish farming [6].

Broadly speaking, differential equations are divided into two main categories:

1. First-Order Differential Equations: This type only involves the first derivative of a function.
2. Higher-Order Differential Equations: This type involves the second or higher derivatives of a function [7].

2.2 Bernoulli Differential Equation

The Bernoulli differential equation is a first-order differential equation that has the form:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n, \quad n \neq 0, 1 \quad (1)$$

with :

y : searched function

$P(x)$ and $Q(y)$: function of the independent variable of x

n : powers of real numbers other than 0 or 1.

The method for solving the Bernoulli equation is usually done by substitution:

$$v = y^{1-n} \quad (2)$$

so that the Bernouli equation can be converted into a linear differential equation which is easier to solve.

2.3 Population Growth Model

Population growth in biology is often modeled using differential equations. The basic models used are:

1. Exponential model

$$\frac{dN}{dt} = rN \quad (3)$$

2. Logistics model

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) \quad (4)$$

3. Bernoulli Model The Bernoulli equation can also be used to model fish production growth by adding non-linear factors, such as resource limitations, competition, and variation.

2.4 Application of Bernoulli's Equation in Catfish Production

In the context of catfish farming, the production level is highly influenced by several factors, including the growth rate of the fish, feed type, stocking density, water quality, and overall environmental conditions. The Bernoulli equation is an effective tool for analyzing fish production growth due to several advantages:

1. It is capable of representing growth rates that are not always straight or linear.
2. It can account for limiting factors, such as the maximum capacity of the cultivation pond.
3. It provides the ability to predict fish production patterns or trends over time.

Therefore, the Bernoulli differential equation can serve as a strong mathematical foundation for analyzing, estimating, and predicting the growth of catfish production under various cultivation conditions.

3. METHODOLOGY

3.1 Types of research

This research is descriptive quantitative with a mathematical modeling approach using the Bernoulli differential equation [2]. The descriptive quantitative approach was chosen to analyze and describe the actual trend of catfish production growth based on existing numerical data [4]. The primary goal is to provide an accurate quantitative depiction of production dynamics and potential carrying capacity (K), and to compare model results with actual data to identify any anomalies [8].

3.2 Data source

The data used in this study is secondary data in the form of annual catfish production (*Clarias* sp.) in Indonesia. The data covers the period from 2019 to 2025. This production data was obtained from the Indonesian Ministry of Marine Affairs and Fisheries Fish Production Statistics Data Portal [3]

3.3 Research Variables

The variables used in this research consist of independent and dependent variables. The independent variable is time (t), which is presented in years [6]. This variable serves as the driver of change in the model. Meanwhile, the dependent variable is catfish production ($N(t)$), measured in tons [9]. This dependent variable represents the actual output being modeled and predicted by the Bernoulli differential equation.

3.4 Data Processing

The catfish production data was collected from the Ministry of Marine Affairs and Fisheries Fish Production Statistics Data Portal. The data was organized into a spreadsheet and verified for any missing or inconsistent entries against official records. The data was then converted into a time series format for use in the Bernoulli differential equation model. The production values were processed and maintained in tons.

3.5 Model Assumptions

The application of the Bernoulli differential equation to catfish production in this study is based on the following assumptions:

1. Constant Environmental Conditions: It is assumed that environmental factors such as water quality and temperature remain relatively constant over the study period.
2. Negligible External Shocks: External shocks such as major disease outbreaks or sudden changes in market demand are not explicitly accounted for in the model.
3. Homogeneous Production Practices: It is assumed that catfish farming practices across Indonesia are relatively uniform

3.6 Research Procedures

1. Collect production data from the KKP portal.
2. Process data in the form of tables and graphs.
3. Form a growth model with the Bernoulli equation.
4. Solving the model analytically. The analytical method was chosen for its precision in determining the model's parameters and for its ability to provide a clear, mathematical solution that accurately fits the catfish production data.
5. Compare model results with actual data and draw conclusions.

3.7 Model Validation

Model validation was conducted through the following methods:

1. Comparison with Actual Data: The model's predictions were compared with actual catfish production data from 2019 to 2025.
2. Statistical Measures: The root mean square error (RMSE) was calculated to quantify the difference between the predicted and actual values.
3. Sensitivity Analysis: A sensitivity analysis was performed to assess the model's robustness by varying key parameters such as the intrinsic growth rate (r) and carrying capacity (K).

4. RESULT & DISCUSSION

4.1 Data Processing Results

Catfish production data in Indonesia from 2019 to 2025 is shown in **Table 1**.

Table 1. Catfish Production in Indonesia (2019–2025)

Year	Production (tons)
2019	1,088,945.18
2020	993,768.27
2021	1,041,422.43
2022	1,101,625.11
2023	1,136,618.98
2024	1,171,367.00
2025	615,909.64

The production graph in **Figure 1** shows a fluctuating trend: a decline in 2020, a gradual increase in 2021-2024, and a sharp decline in 2025.

4.2 Modeling with Bernoulli's Differential Equation

Bernoulli's equation takes the form:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n, \quad n \neq 0, 1$$

With substitution $v = y^{1-n}$, the equation can be transformed into a linear differential equation which is easier to solve. In the context of biology and production, a special form with $n = 2$ generate a logistics model :

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right)$$

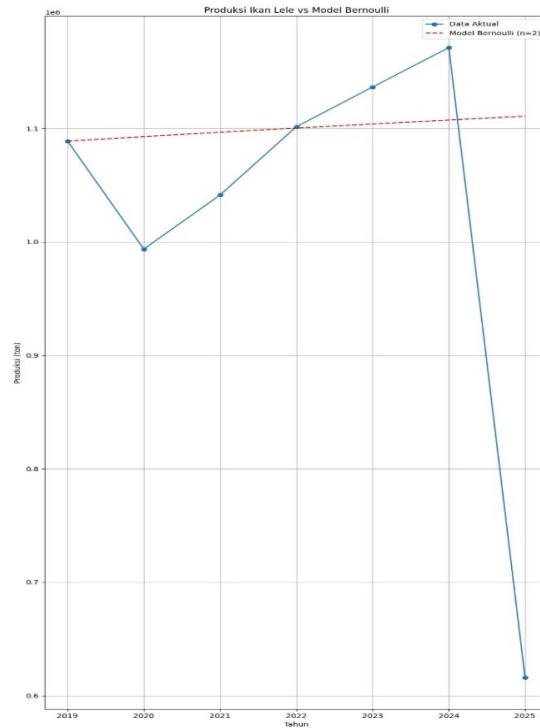


Figure 1. Catfish Production vs Bernoulli's Model.

with general solutions:

$$N(t) = \frac{k}{1 + Ae^{-rt}}$$

where :

$N(t)$: catfish production in year t

k : maximum capacity (carrying capacity)

r : intrinsic growth rate

A : constant of initial conditions

4.3 Model Parameter Estimation

Based on the 2019-2024 data:

1. The maximum capacity (carrying capacity) is estimated as $K \approx 1,200,000$ tons.
2. The average intrinsic growth rate is estimated as ≈ 0.04 (4% per year)
3. With the initial condition $N(0) = 1.088.945,18$ tons (2019 production), the constant A is obtained as $A \approx 0.10$

Thus, the catfish production growth model derived from the Bernoulli equation (in its logistic form) can be written as:

$$N(t) = \frac{1200000}{1 + 0,10 e^{-0,04t}}$$

4.4 Comparison of Model and Actual Data

1. 2020-2024: Model results are quite close to actual data, indicating a realistic logistics growth pattern. Production is increasing close to maximum capacity.

2. 2025: The model predicts production of around 1.19 million tonnes, while actual data is only 615 thousand tonnes. This large difference indicates the presence of external factors that are not included in the model. (for example fish disease, decreased water quality, or distribution disruptions).

4.5 Data Trend Analysis

1. 2019-2020: Production fell by about 8.7%, likely due to the impact of the COVID-19 pandemic.
2. 2020-2024: Production increases steadily with an average growth of 4% per year, reflecting improvements in cultivation management.
3. 2025: A sharp decline of around 47% indicates serious constraints in production.

4.6 Discussion

4.6.1 Discussion on Model Interpretation and Contribution

The results show that the Bernoulli/logistic model is able to explain the normal growth pattern between 2020 and 2024 quite well²⁹. This implies that catfish production growth is not exponential, but is effectively limited by environmental factors and carrying capacity (K). The model provides a realistic mathematical basis for understanding the dynamics of production growth under stable conditions.

4.6.2 Analysis of the 2025 Anomaly and Contribution

The significant mismatch observed in 2025, where the actual production dropped sharply to 615 thousand tonnes against the model's prediction of 1.19 million tonnes, highlights the study's key contribution: The model functions as an anomaly detection tool. This sharp decline (around 47%) indicates the strong influence of external non-linear factors that were not accounted for in the pure mathematical framework. Specifically, the decline could be attributed to serious constraints such as disease outbreaks (e.g., *Aeromonas hydrophila* infections), environmental stress (deterioration of water quality), or distribution disruptions.

4.6.3 Managerial Implications

These findings underline that purely mathematical models, while valuable, must be complemented with ecological and managerial analyses to fully capture the complexity of aquaculture systems. This serves as crucial information for management strategies related to disease prevention, environmental monitoring, and supply chain stabilization to mitigate future production declines

5. CONCLUSION

This study concludes that Bernoulli's differential equation, particularly in its logistic form, is effective for modeling the growth of catfish production in Indonesia. The model successfully explains the increasing trend observed from 2020 to 2024, showing that catfish production tends to stabilize near its carrying capacity rather than growing exponentially. However, the significant deviation observed in 2025 reveals that external non-mathematical factors such as environmental quality, disease outbreaks, and cultivation management practices strongly influence production outcomes. These findings underline that while mathematical models provide valuable insights into growth dynamics, they must be complemented with ecological and managerial analyses to fully capture the complexity of aquaculture systems. Future research could integrate environmental and socioeconomic variables into the model to improve predictive accuracy and provide more comprehensive guidance for sustainable catfish farming.

REFERENCES

- [1] E. Nurhayati, "Kajian pemanfaatan fish processing guna mewujudkan ketahanan pangan wilayah perbatasan indonesia–timor leste," in *Seminar Nasional Kontribusi Vokasi*, vol. 1, no. 1, Mar. 2024, pp. 338–344.
- [2] D. Jatnika, K. Sumantadinata, and N. H. Pandjaitan, "Pengembangan usaha budidaya ikan lele (*clarias* sp.) di lahan kering di kabupaten gunungkidul, provinsi daerah istimewa yogyakarta," *Manajemen IKM: Jurnal Manajemen Pengembangan Industri Kecil Menengah*, vol. 9, no. 1, pp. 96–105, 2014.

- [3] Kementerian Kelautan dan Perikanan, “Laporan kinerja 2024,” 2024, menjelaskan produksi perikanan budidaya nasional.
- [4] M. F. Taufiq, N. D. Yanti, and A. Rahman, “Analisis dampak ekonomi dan sosial pada lingkungan hidup terhadap usaha ikan lele di desa basanah dalam perspektif hifd al biah,” *Jurnal Ekonomi, Bisnis dan Manajemen*, vol. 4, no. 2, pp. 278–286, 2025.
- [5] M. S. Rahmi and Y. Haryono, *Buku Ajar Persamaan Diferensial Biasa*. Deepublish, 2020.
- [6] N. V. Natasya, N. Permadani, I. Ikawati, and K. Kurniawan, “Pendekatan matematika yang digunakan pada biologi,” *Populer: Jurnal Penelitian Mahasiswa*, vol. 3, no. 4, pp. 73–84, 2024.
- [7] M. Misbah, *Persamaan Differensial Matematika Fisika*, 2022.
- [8] M. Z. Ndi, *Pemodelan Matematika Dinamika Populasi dan Penyebaran Penyakit: Teori, Aplikasi, dan Numerik*. Deepublish, 2018.
- [9] E. Herlinawati and A. Khotimah, “Analisis pertumbuhan ikan lele dengan model von bertalanffy di kolam peternak, desa mekarjaya, kuningan, jawa barat,” *Teorema: Teori dan Riset Matematika*, vol. 7, no. 1, pp. 65–76, 2022.