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Simplified Computation Method of Useful Functions in the Offset Fractional Fourier Transform

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ABSTRACT
The offset fractional Fourier transform (OFrFT) may be interpreted as a generalization of the classical Fourier transform since the Fourier transform itself represents a particular case of the linear canonical transform. Due to this relationship, the OFrFT has become a useful tool in signal analysis. Several essential properties of the Fourier transform can be extended to the OFrFT domain with appropriate modifications. In this paper, we investigate the analytical connection between the offset fractional Fourier transform and the classical Fourier transform. Based on this relationship, an efficient method for evaluating the Gaussian function in the OFrFT domain is developed. To demonstrate the applicability of the proposed approach, several examples involving Gaussian functions in the OFrFT domain are presented.

KEYWORDS
Fourier Transform, Gaussian Function, Offset Fractional Fourier Transform.

1. INTRODUCTION

The offset fractional Fourier transform (OFrFT) is an extension of the classical Fourier transform (FT) that has attracted considerable attention in signal analysis and related fields. As a generalized transform, the OFrFT introduces additional parameters that provide greater flexibility in representing signals in transformed domains. Because of these characteristics, the OFrFT has become a useful mathematical tool in several applications, including signal processing, optical systems, time–frequency analysis, encryption, modulation, and communication systems (see [1, 2, 3, 4, 5, 6], [7]).

Many important properties of the classical Fourier transform can be extended to the OFrFT domain with suitable modifications. These properties enable the OFrFT to play an essential role in the analysis and processing of signals in more generalized transform domains. Several theoretical properties and applications of related transforms have been investigated in the literature ([8, 9, 10], [11], [12]).

In this paper, we investigate the analytical connection between the OFrFT and the classical FT. Based on this relationship, a simplified computational method is developed to evaluate Gaussian functions in the OFrFT domain. Several examples are presented to demonstrate the effectiveness of the proposed approach.

2. LITERATURE REVIEW

In this section, the definition of the OFrFT together with its inverse transform is presented.

Definition 2.1. [9, 13, 11] Let $f \in L^1(\mathbb{R})$. The OFrFT of a function f is defined by

$$\mathscr{F}_{\omega}^{(\gamma,p,q)}\{f\}(\eta) = \int_{\mathbb{R}} f(z) K_{(\gamma,p,q)}(z,\eta)\, dz \tag{1}$$

where the kernel function $K_{(\gamma,p,q)}(z,\eta)$ is given by

$$K_{(\gamma,p,q)}(z,\eta) = A_\gamma e^{\frac{i}{2}p^2\cot\gamma} e^{\frac{i}{2}(\eta^2+z^2)\cot\gamma+2z(p-\eta)\csc\gamma+2\eta(q-p)\cot\gamma} \tag{2}$$

where

$$A_\gamma = \sqrt{\frac{1-i\cot\gamma}{2\pi}}.$$

Here, the parameters (γ,p,q) are real numbers with the condition $\gamma \neq n\pi$, where $n \in \mathbb{Z}$. Moreover, the original signal $f(z)$ can be recovered using the inverse formula of the OFrFT, which is given as follows.

Theorem 2.1. [9, 13, 11] For every $f \in L^1(\mathbb{R})$ and $\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\} \in L^1(\mathbb{R})$, the inverse of the OFrFT is given by

$$f(z) = \int_{\mathbb{R}} \mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\, \overline{K_{(\gamma,p,q)}(z,\eta)}\, d\eta. \tag{3}$$

3. MAIN RESULT

In this section, the Gaussian signal in the OFrFT domain is evaluated by exploiting the analytical connection between the FT and the OFrFT. To proceed, we first recall the definition of the FT (see [13]).

Definition 3.1. [13]. Let $f \in L^1(\mathbb{R})$. The FT of f is defined by the integral representation

$$\mathscr{F}\{f\}(\eta) = \hat{f}(\eta) = \int_{\mathbb{R}} f(z) e^{-i\eta z} dz \tag{4}$$

Using (1), the definition of the OFrFT can be reformulated as the classical FT, namely,

$$\begin{aligned}
\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta) =& A_\gamma \int_{\mathbb{R}} f(z) e^{\frac{i}{2}p^2\cot\gamma} e^{\frac{i}{2}((\eta^2+z^2)\cot\gamma+2z(p-\eta)\csc\gamma+2\eta(q-p\cot\gamma))}\, dz \\
=& A_\gamma e^{\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))} \int_{\mathbb{R}} f(z) e^{\frac{i}{2}(z^2\cot\gamma+2zp\csc\gamma)-i\eta\csc\gamma}\, dz \\
=& A_\gamma e^{\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))}\, \mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right)
\end{aligned} \tag{5}$$

Denoted by

$$f_\gamma(z) = f(z) e^{\frac{i}{2}(z^2\cot\gamma+2zp\csc\gamma)}. \tag{6}$$

Then we obtain

$$\mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right) = \sqrt{\frac{2\pi}{1-i\cot\gamma}}\, \mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\, e^{-\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))}. \tag{7}$$

Using the relation obtained above, the Gaussian signal can be evaluated in the OFrFT domain. The Gaussian function is expressed in the following form:

$$f(z) = e^{-az^2} \tag{8}$$

From the FT in the Definition 3.1, it follows that

$$\begin{aligned}\mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right) &= \mathscr{F}\left\{f(z)e^{\frac{i}{2}(z^2\cot\gamma+2zp\csc\gamma)}\right\}\left(\frac{\eta}{\sin\gamma}\right)\\ &= \mathscr{F}\left\{e^{-az^2}e^{\frac{i}{2}(z^2\cot\gamma+2zp\csc\gamma)}\right\}\left(\frac{\eta}{\sin\gamma}\right)\end{aligned} \tag{9}$$

By definition of the FT defined by equation (4), then equation can be rewritten as

$$\begin{aligned}\mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right) &= \int_{\mathbb{R}} e^{-az^2}e^{\frac{i}{2}(z^2\cot\gamma+2zp\csc\gamma)}e^{-iz\frac{\eta}{\sin\gamma}}\,dz\\ &= \int_{\mathbb{R}} e^{-az^2}e^{\frac{i}{2}(z^2\cot\gamma+2zp\csc\gamma)}e^{-iz\frac{\eta}{\sin\gamma}}\,dz\\ &= \int_{\mathbb{R}} e^{-\left(a-\frac{i\cos\gamma}{2\sin\gamma}\right)z^2+i\left(\frac{p-\eta}{\sin\gamma}\right)z}dz\\ &= \int_{\mathbb{R}} e^{-\left(a-\frac{i\cos\gamma}{2\sin\gamma}\right)\left(z^2-2i\left(\frac{p-\eta}{2a\sin\gamma-i\cos\gamma}\right)z\right)}\,dz\\ &= \int_{\mathbb{R}} e^{-\left(a-\frac{i\cos\gamma}{2\sin\gamma}\right)\left(\left(z-i\left(\frac{p-\eta}{2a\sin\gamma-i\cos\gamma}\right)\right)^2+\left(\frac{p-\eta}{2a\sin\gamma-i\cos\gamma}-\frac{p-\eta}{2a\sin\gamma-i\cos\gamma}\right)^2\right)}dz\\ &= \int_{\mathbb{R}} e^{-\left(a-\frac{i\cos\gamma}{2\sin\gamma}\right)\left(z-i\left(\frac{p-\eta}{2a\sin\gamma-i\cos\gamma}\right)\right)^2}dz\,e^{-\frac{(p-\eta)^2}{2\sin\gamma(2a\sin\gamma-i\cos\gamma)}}\end{aligned} \tag{10}$$

Now putting

$$P=\left(a-\frac{i\cos\gamma}{2\sin\gamma}\right),\quad y=\left(z-i\left(\frac{p-\eta}{2a\sin\gamma-i\cos\gamma}\right)\right) \tag{11}$$

Then, we get

$$\mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right)=\int_{\mathbb{R}} e^{-Py^2}\,dz\,e^{-\frac{(p-\eta)^2}{2\sin\gamma(2a\sin\gamma-i\cos\gamma)}} \tag{12}$$

Hence,

$$\mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right)=e^{-\frac{(p-\eta)^2}{2\sin\gamma(2a\sin\gamma-i\cos\gamma)}}\sqrt{\frac{\pi}{P}} \tag{13}$$

Substituting the first term of relation (6) into relation (7), one could easily obtain

$$\mathscr{F}\{f_\gamma\}\left(\frac{\eta}{\sin\gamma}\right)=e^{-\frac{(p-\eta)^2}{2\sin\gamma(2a\sin\gamma-i\cos\gamma)}}\sqrt{\frac{2\pi\sin\gamma}{2a\sin\gamma-i\cos\gamma}} \tag{14}$$

It follows from relation (5) that

$$\begin{aligned}&\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\\ &=\sqrt{\frac{1-i\cot\gamma}{2\pi}}\sqrt{\frac{2\pi\sin\gamma}{2a\sin\gamma-i\cos\gamma}}e^{-\frac{(p-\eta)^2}{2\sin\gamma(2a\sin\gamma-i\cos\gamma)}}e^{\frac{i}{2}\left((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma)\right)}\\ &=\sqrt{\frac{(\sin\gamma-i\cos\gamma)}{(2a\sin\gamma-i\cos\gamma)}}e^{-\frac{(p-\eta)^2}{2\sin\gamma(2a\sin\gamma-i\cos\gamma)}}e^{\frac{i}{2}\left((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma)\right)}\\ &=\sqrt{\frac{(\sin\gamma-i\cos\gamma)(2a\sin\gamma+i\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}e^{-\frac{(p-\eta)^2(2a\sin\gamma+i\cos\gamma)}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}e^{\frac{i}{2}\left((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma)\right)}\end{aligned} \tag{15}$$

Therefore,

$$\begin{aligned}
&\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\\
&=\sqrt{\frac{(2a\sin^2\gamma+\cos^2\gamma)+i(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}\,e^{-\frac{(p-\eta)^2(2a\sin\gamma+i\cos\gamma)}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}\,e^{\frac{i}{2}(\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma)}.
\end{aligned} \tag{16}$$

The above expression shows that the obtained equation agrees with the Gaussian function in the OFrFT domain. The behavior of the Gaussian signal in the OFrFT domain can be analyzed for different parameter values. Furthermore, the norm (or modulus) of the Gaussian signal in the OFrFT domain is evaluated. Hence, the following expression is obtained.

$$\begin{aligned}
&\left|\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\right|\\
&=\sqrt{\frac{(2a\sin^2\gamma+\cos^2\gamma)+i(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}\cdot e^{-\frac{(p-\eta)^2(2a\sin\gamma+i\cos\gamma)}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}\,e^{\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))}\\
&=\left|\sqrt{\frac{(2a\sin^2\gamma+\cos^2\gamma)+i(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}\right|\left|e^{-\frac{(p-\eta)^2(2a\sin\gamma+i\cos\gamma)}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}\right|\left|e^{\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))}\right|\\
&=\left|\sqrt{\frac{(2a\sin^2\gamma+\cos^2\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}+\frac{i(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}\right|\left|e^{-\frac{(p-\eta)^2 2a\sin\gamma}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}+i\frac{\cos\gamma}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}\right|\\
&\quad\times\left|e^{\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))}\right|\\
&=\left|\sqrt{\frac{(2a\sin^2\gamma+\cos^2\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}+\frac{i(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}\right|\left|e^{-\frac{(p-\eta)^2 2a\sin\gamma}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}\right|\left|e^{i\frac{\cos\gamma}{2\sin\gamma(4a^2\sin^2\gamma+\cos^2\gamma)}}\right|\\
&\quad\times\left|e^{\frac{i}{2}((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma))}\right|\\
&=\left|\sqrt{\frac{(2a\sin^2\gamma+\cos^2\gamma)+i(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)}{4a^2\sin^2\gamma+\cos^2\gamma}}\right|e^{-\frac{a(p-\eta)^2}{(4a^2\sin^2\gamma+\cos^2\gamma)}}
\end{aligned} \tag{17}$$

Observe that

$$(a+ib)^{\frac{1}{2}}=\left(\sqrt{a^2+b^2}e^{i\arctan\left(\frac{b}{a}\right)}\right)^{\frac{1}{2}}=(a^2+b^2)^{\frac{1}{4}}e^{\frac{i}{2}\arctan\left(\frac{b}{a}\right)} \tag{18}$$

Since $|e^{i\theta}|=1$, therefore

$$\begin{aligned}
\left|(a+ib)^{\frac{1}{2}}\right|&=\left|(a^2+b^2)^{\frac{1}{4}}e^{\frac{i}{2}\arctan\left(\frac{b}{a}\right)}\right|\\
&=(a^2+b^2)^{\frac{1}{4}}
\end{aligned}$$

Applying $\left|(a+ib)^{\frac{1}{2}}\right|=(a^2+b^2)^{\frac{1}{4}}$ in the above equation (17), we get

$$\left|\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\right|=\left(\frac{(2a\sin^2\gamma+\cos^2\gamma)^2+(\sin\gamma\cos\gamma-2a\sin\gamma\cos\gamma)^2}{(4a^2\sin^2\gamma+\cos^2\gamma)^2}\right)^{\frac{1}{4}}e^{-\frac{a(p-\eta)^2}{(4a^2\sin^2\gamma+\cos^2\gamma)}}. \tag{19}$$

The obtained result represents the norm (modulus) of the Gaussian function in the Fourier transform domain. Figure 1 shows the behavior of the Gaussian signal in the OFrFT domain for several values of γ.

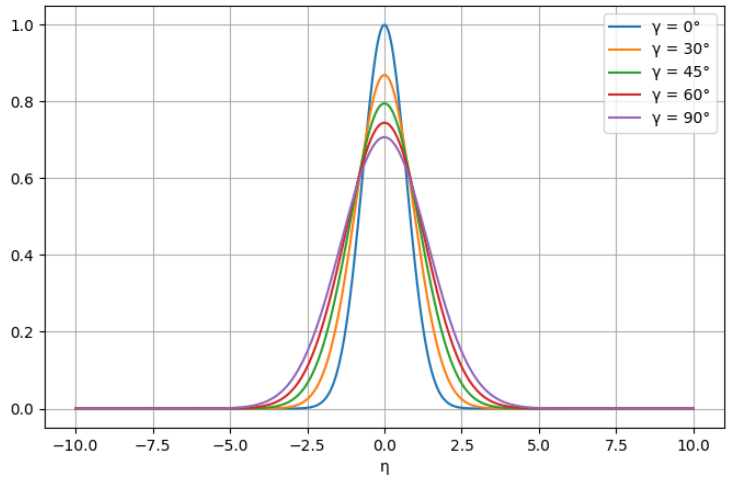


Figure 1. Gaussian signal norm in the OFrFT domain for different values of γ.

In the final step, the established relationship is used to determine the following function within the OFrFT domain. The function is expressed as:

$$f(z) = \begin{cases} 2, & -2 \le z \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

In fact, we have

$$\begin{aligned} \mathscr{F}\{f_y\}(\eta) &= \int_{\mathbb{R}} f_y(z) e^{-i\eta z}\, dz \\ &= \int_{\mathbb{R}} f(z) e^{\frac{i}{2}(z^2 \cot\gamma + 2zp\csc\gamma)} e^{-i\eta z}\, dz \\ &= \int_{\mathbb{R}} f(z) e^{\frac{i}{2}(z^2 \cot\gamma + 2z(p\csc\gamma - \eta))}\, dz \\ &= 2\int_{\mathbb{R}} e^{\frac{i}{2}\cot\gamma\left(\left(z + \frac{(p\csc\gamma-\eta)}{\cot\gamma}\right)^2 - \frac{(p\csc\gamma-\eta)^2}{\cot\gamma}\right)} dz \\ &= 2e^{\frac{i(p\csc\gamma-\eta)^2}{2\cot\gamma}} \int_{-2}^{2} e^{\frac{i}{2}\cot\gamma\left(iz + \frac{i(p\csc\gamma-\eta)}{\cot\gamma}\right)^2} dz \end{aligned} \tag{20}$$

Letting $u = iz + \frac{i(p\csc\gamma-\eta)}{\cot\gamma}$, we further obtain

$$\begin{aligned} \mathscr{F}\{f_y\}(\eta) &= 2e^{-\frac{i(p\csc\gamma-\eta)^2}{2\cot\gamma}} \int_{-2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}}^{2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}} e^{-\frac{i}{2}\cot\gamma u^2} \frac{du}{i} \\ &= -2ie^{\frac{i(p\csc\gamma-\eta)^2}{2}} \cot\gamma - \frac{i\pi}{2} \int_{-2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}}^{2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}} e^{-\left(\sqrt{\frac{i}{2}\cot\gamma}u\right)^2} du \end{aligned} \tag{21}$$

Let

$$y = \sqrt{\frac{i}{2}\cot\gamma}\, u, \qquad \frac{dy}{\sqrt{\frac{i}{2}\cot\gamma}} = du$$

Hence,

$$\mathscr{F}\{f_y\}(\eta) = -2ie^{-\frac{i(p\csc\gamma-\eta)^2}{2\cot\gamma}-i\frac{\pi}{2}} \int_{\sqrt{\frac{i}{2}\cot\gamma}\left(-2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}\right)}^{\sqrt{\frac{i}{2}\cot\gamma}\left(2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}\right)} e^{-y^2}\frac{dy}{\sqrt{\frac{i}{2}\cot\gamma}}$$

Using the fact,

$$\int_0^a e^{-z^2}\,dz = \frac{\sqrt{\pi}}{2}\operatorname{erf}(a)$$

Therefore,

$$\begin{aligned}\mathscr{F}\{f_y\}(\eta) =& ie^{-\frac{i(p\csc\gamma-\eta)^2}{2\cot\gamma}-i\frac{\pi}{2}}\frac{\sqrt{2\pi}}{\sqrt{i\cot\gamma}} \\ &\times\left(\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(2i+\frac{i(p\csc\gamma-\eta)}{c\cot\gamma}\right)\right)-\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(-2i+\frac{i(p\csc\gamma-\eta)}{\cot\gamma}\right)\right)\right)\end{aligned}$$

Due to the scaling property in the frequency-domain representation, the frequency variable undergoes a transformation involving the factor $\sin\gamma$. Therefore, to maintain consistency of the expression, we introduce the substitution η with $\frac{n}{\sin\gamma}$ in (22), we get

$$\begin{aligned}&\mathscr{F}\{f_y\}\left(\frac{\eta}{\sin\gamma}\right) \\ =&ie^{-\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)^2}{2\cot\gamma}-i\frac{\pi}{2}} \\ &\frac{\sqrt{2\pi}}{\sqrt{i\cot\gamma}}\left(\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(2i+\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)}{\cot\gamma}\right)\right)-\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(-2i+\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)}{\cot\gamma}\right)\right)\right)\end{aligned} \tag{22}$$

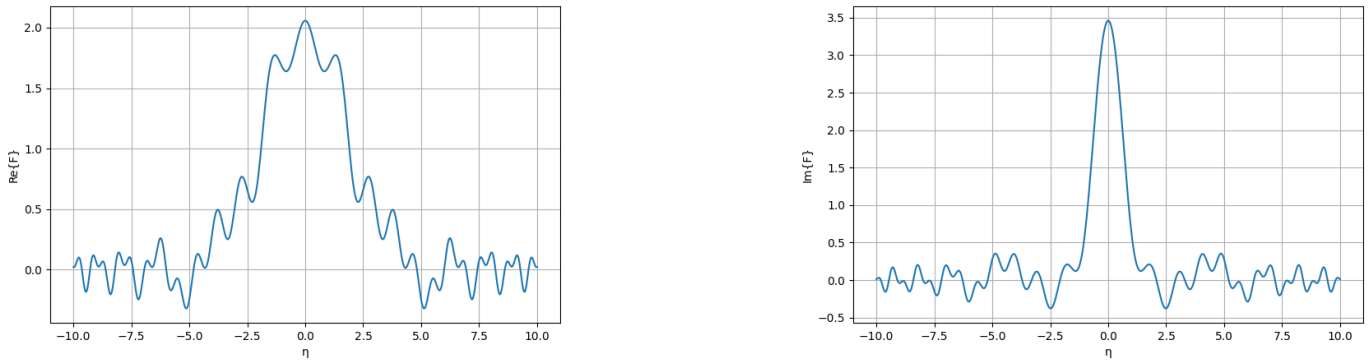
With the help of equation (7), we arrive at

$$\begin{aligned}&\sqrt{\frac{2\pi}{1-i\cot\gamma}}\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta)\,e^{-\frac{i}{2}\left((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma)\right)} \\ =&ie^{-\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)^2}{2\cot\gamma}-i\frac{\pi}{2}} \\ &\frac{\sqrt{2\pi}}{\sqrt{i\cot\gamma}}\left(\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(2i+\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)}{\cot\gamma}\right)\right)-\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(-2i+\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)}{\cot\gamma}\right)\right)\right)\end{aligned} \tag{23}$$

Or equivalently,

$$\begin{aligned}&\mathscr{F}_{\mathbb{O}}^{(\gamma,p,q)}\{f\}(\eta) \\ =&i\sqrt{\frac{1-i\cot\gamma}{\cot\gamma}}e^{-\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)^2}{2\cot\gamma}-i\frac{\pi}{2}+\frac{i}{2}\left((\eta^2+p^2)\cot\gamma+2\eta(q-p\cot\gamma)\right)} \\ &\left(\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(2i+\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)}{\cot\gamma}\right)\right)-\operatorname{erf}\left(\sqrt{\frac{i}{2}\cot\gamma}\left(-2i+\frac{i\left(p\csc\gamma-\frac{\eta}{\sin\gamma}\right)}{\cot\gamma}\right)\right)\right)\end{aligned} \tag{24}$$

The graphical representation of Equation (24), including its real and imaginary parts, is shown in Figure 2



(a) (i) Real part **(b)** (ii) Imaginary part

Figure 2. Real and imaginary parts of Equation (3.19): (i) real part, (ii) imaginary part.

4. CONCLUSION

This paper presents a method for computing Gaussian and piecewise functions based on the relationship between the OFrFT and the conventional Fourier transform. The obtained results are supported by simulations in the OFrFT domain. This approach offers a foundation for extending the analysis to more general functions within the OFrFT framework in future studies.

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